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DIGITAL COMPUTER APPLICATION TO PROCESS CONTROL PROBLEMS

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled DIGITAL COMPUTER APPLICATION TO PROCESS CONTROL PROBLEMS by J.E. Eric Lofkrantz, B.Sc. in partial fulfilment of the requirements for the degree of Master of Science in Chemical Engineering.

ABSTRACT

This thesis presents the results of an evaluation and extension of an I.B.M. digital computer program designed for control systems analysis.

The original program proved useful in the determination of root locus diagrams, and the modified program in the determination of Bode plots, Nyquist plots, and transient response of control systems.

The transient response calculation uses the Z-transform ability of the original program plus a power series inversion. This method is a modified form of the Boxer and Thaler method(1), and is used in the belief that it provides more overall flexibility.

The transient response of continuous systems can be determined using this program, if a fictitious sample and hold device is added to the system input. A lead is also added to counteract the lag introduced by the sample and hold device. Thus, for the continuous system, the Z-transform of the product of the closed loop transfer function, the sample and hold transfer function, and the fictitious lead is calculated. This Z-transfer function is then inverted to obtain points on the transient response curve. Sampled-data systems are analyzed directly without the addition of any fictitious lead or hold device.

Seven examples are presented to illustrate the manner in which the program can be employed to handle different control problems in the design or analysis such as:

1. the effect of pure time delay;
2. the effect of sampling an analog control system;
3. the design of digital controllers; and
4. the design of analog controllers.

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1. INTRODUCTION

This thesis consists primarily of the results of an evaluation and extension of an I.B.M. digital computer program for use in Control Systems Analysis. The investigation was to determine the program's capabilities and limitations as applied to chemical engineering process control problems as well as examine the program for possible extension.

It was thought that the program should satisfy the following conditions:

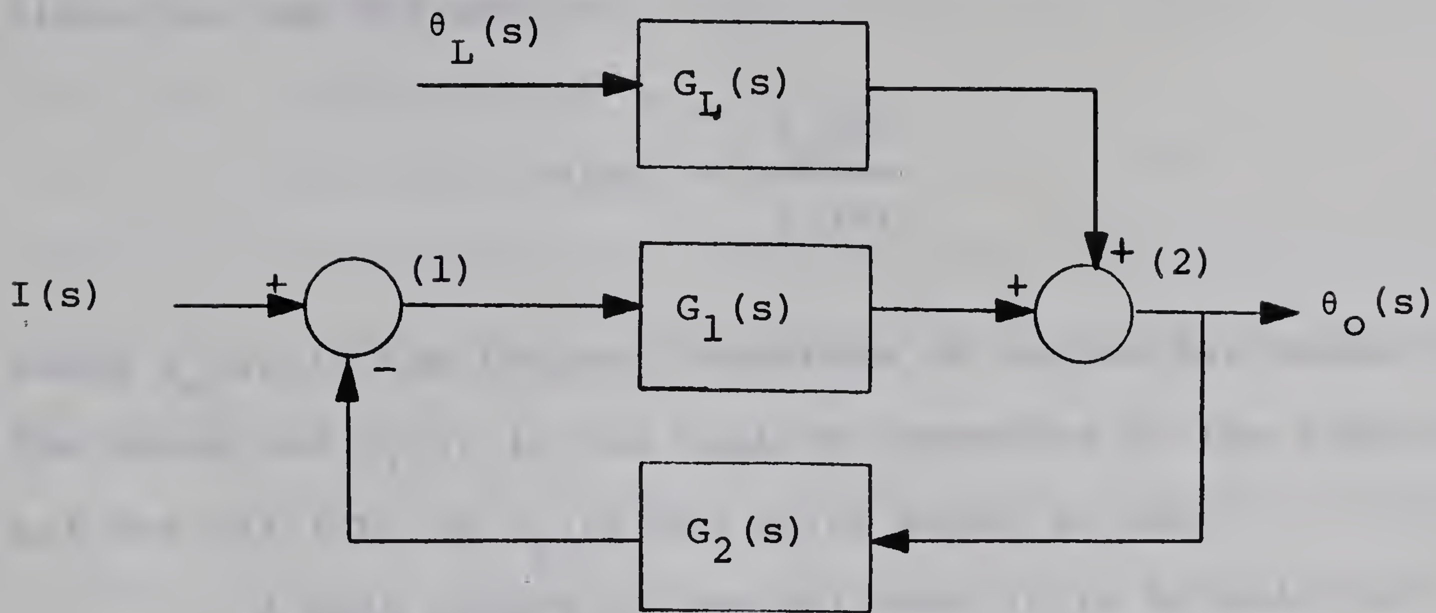
1. It should provide a significant contribution to the overall systems analysis.
2. It should be user orientated.
3. The outcome of the analysis should be presented in the most easily understood form.
4. Program options should be user specified so that the analysis can be tailored to the user's specific problem.
5. Computation time should be kept to a minimum.

A controlled system can be represented very simply by the use of block diagrams. A simple feedback control system, in block diagram form, is shown in Figure 1, primarily as an introduction to the terminology used throughout this thesis.

Referring to Figure 1, the part of the control system containing $G_1(s)$, that is the system between 1 and 2 in Figure 1, will subsequently be designated as the "forward loop" of the

Figure 1

General Block Diagram



Nomenclature

$I(s)$	Setpoint Variable
$\theta_o(s)$	Output Variable
$\theta_L(s)$	Load Variable

Math Model

$$\left[1 + G_1(s)G_2(s)\right]\theta_o(s) = G_1(s)\theta_i(s) + G_L(s)\theta_L(s)$$

Characteristic Equation

$$1 + G_1(s)G_2(s) = 0$$

system. A path such as the one containing $G_2(s)$ and extending from 2 to 1 will be designated as a "feedback loop". $G_1(s)$, $G_2(s)$, and $G_L(s)$ will be referred to as transfer functions and are defined as:

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)}$$

where $\theta_o(s)$ is the Laplace transform of the output signal of the block and $\theta_i(s)$ is the Laplace transform of the input signal for all I.C. of $\theta_o(t)$ and $\theta_i(t)$ equal to zero.

A main object in control work is to be able to analyze a system such as that shown in Figure 1, for its dynamic characteristics. From this study, information necessary to determine:

1. The type of controller
2. The control parameter settings

should be derived so that

1. The stability of the system is satisfactory
2. The control criteria are met.

A method commonly used for analyzing a system for stability and the one upon which this control system analysis program is based is the Root Locus Technique. A Root Locus is defined to be the trajectory in the "s"-plane followed by a root of the characteristic equation as some parameter of the corresponding system is varied continuously. The

parameter which is varied in this analysis is the overall loop gain K. In Figure 1 the characteristic equation for the system is $1 + G_1(s)G_2(s) = 0$ where K is included in one of the transfer functions, but is usually separated for the purpose of a root locus analysis.

This characteristic equation determines the transient response of a system to any input and thus the location of its roots can be used to indicate stability or instability for given values of K.

Analysis of control systems using frequency response or root locus techniques has generally been a laborious task and because of this a really complete investigation is seldom done. With the digital computer it is now possible to greatly reduce the time expended in performing these analyses and thus, greatly increase the extent of their application. The ultimate, of course, is to have the analysis and design calculations performed completely by the machine with only the subjective decisions left to the user.

The investigation was conducted with this ultimate goal in mind.

2. LITERATURE SURVEY

This thesis deals with the role of the digital computer in the formation and use of the various control system stability analysis aids, such as the Bode, Nyquist, Root Locus and Transient Response curves.

Texts found to be of particular use in outlining the basic theory behind the Root Locus, Nyquist and Bode Plot methods of analysis were those by Evans(4) and Schilling(18).

A summary of papers dealing with frequency response methods is presented by Oldenburger(16). This is a collection of basic papers in the field and provides an excellent list of references for further search.

The data derived from frequency response studies consists of different values of amplitude ratio and phase angle between the input and output of a sinusoidally forced system for different frequencies. This type of data lends itself to analysis by the Bode and Nyquist methods but it cannot be used, as is, in a Root Locus analysis, since this type of analysis depends on having the transfer function of the system. Some methods have been worked out to obtain the transfer functions for certain systems from this type of data. A graphical method is presented by Ganapathy and Krishna(6) and a numerical method by Staffin and Staffin(20). An analytical method for obtaining a root locus of a given characteristic equation is outlined by Chang(2), and the analogue computer is used in conjunction with

the generalized Mitrovic method in a procedure outlined by Kokotovic and Siljak(11).

The above methods are means of analyzing a system for its transient response characteristics without actually arriving at the true transient response solution. Wolfang, Wagner and Zoss(22) have presented a method whereby the transient response of a system is obtained from frequency response plots using discrete point data. This method appears suitable for digital computer application. A means of finding the transient response of a system given the input using infinite matrices in time and sampled-data theory is outlined by Dorf(3). This method is particularly suited to digital computer applications as it involves matrix inversion, addition, and multiplication.

Digital computers are beginning to make themselves felt as control instruments, for their versatility as well as their capability of handling a large number of control loops. A discussion of the advantages and disadvantages of Direct Digital Control, the economic considerations, and minimum sampling periods found to be effective for certain types of systems is presented by Klock and Schoeffler(10).

With the digital computer arrives the so-called sampled-data system, and the extension of the methods applied to the continuous system, specifically the root locus method, involves the theory of the Z-transform. An outgrowth of this

is the use of the Z-transform in predicting the transient response of a system to a disturbance. Work has been done along this line by Fowler(5) and by Boxer and Thaler(1).

To use the Z-transform, a basic knowledge of it is necessary and this can be gained from texts by Jury(9), Lindorff(13), and Kuo(12).

Monroe(14) has published a text which is primarily concerned with the synthesis of digitally controlled systems. The various factors which aid in the selection and design of a digital controller are covered. Tou(21) in his paper approaches the design of a "dead-beat" controller for a sampled-data system using the state-variable technique. A complete analysis of a sampled-data system using the Z-transform is given in a paper by Slaughter(19). The analysis is very complete and shows the use of the Z-plane root locus in design.

Some actual control problems, test equipment and the methods used to come to a solution are presented by Rock(17). This is generally a review of testing equipment and the information which can be gained from each. Murrill and Smith(15) have written a paper on the importance of correct controller settings and some present rules of thumb used to arrive at these settings.

3. Z-TRANSFORM THEORY

3.1 The Z-Transform

The Control Systems Analysis Program(7) was originally designed for use as a convenient method for obtaining a root locus in either the s or Z-plane. This capability allows its use in analyzing various types of control systems such as some sampled-data systems, systems with pure time delay, linear systems, or a combination of pure time delay and sampled-data, the latter system containing a pure time delay along with a sampling device. One of the objectives of the design of this program was to have it analyze sampled-data systems and pure time delay systems. To do this, a routine to convert an s-transfer function to its corresponding Z-transfer function was worked out and included in the program.

There are a number of classical methods by which a Z-transform can be calculated from the Laplace Transform, such as the method of complex convolution, by partial fractions where the function is broken down into partial fraction identities which can then be transformed directly to the Z-form, or by the direct numerical evaluation of the series,

$$G(Z) = \left[\frac{1}{T} \sum_{n=-\infty}^{n=\infty} G(s+inw_o)\right]_{Z=e^{sT}} + \frac{1}{2}\, e(0^+) \quad (1)$$

where w_o is the sampling frequency.

The computation required for a Z-transform of a complex system by any of the above becomes very tedious, even for

machine computation where applicable. Because of this, a different method(8) of computing the Z-transform from an s-transform was incorporated into this program by I.B.M., which is particularly suited to machine calculation.

If the degree of s in the numerator of the transfer function G(s) is two or more less than that of the denominator, equation (1) can be written as

$$G(Z) = \left[\frac{1}{T} \sum_{n=-\infty}^{n=+\infty} G(s + inw_o)\right]_{Z=e^{sT}} \quad (2)$$

or in another form

$$\frac{N(Z)}{D(Z)} = \left[\frac{1}{T} \sum_{n=-\infty}^{n=+\infty} G(s + inw_o)\right]_{Z=e^{sT}} \quad (3)$$

The order of the denominator of G(Z) must be the same as that of the denominator of G(s). This correspondence is realized from the conformal mapping of the left hand side of the s-plane into a unit circle centered at the origin in the Z-plane when the transformation $Z = e^{sT}$ is used. This transformation only maps the poles onto a different plane so that the number of roots remains unchanged. Thus, D(Z), which is the denominator of the Z-transfer function can easily be computed by a direct transformation of the poles of G(s) in the s-plane onto the Z-plane. With D(Z) known, equation (3) can be written in the form

$$N(Z) = \frac{D(Z)}{T} \left[\sum_{n=-\infty}^{n=+\infty} G(s + inw_o)\right]_{Z=e^{sT}} \quad (4)$$

where N(Z) is an unknown Z-polynomial and is assumed to be of the same degree as D(Z). The order of N(Z) cannot be greater than that of D(Z) for a physically realizable system. However, N(Z) may be of a lesser degree than D(Z) and for this case, the coefficients for those assumed powers of Z which do not exist will turn out to be equal to zero in the computer program solution.

For any given complex value of s, the complex function represented by the right hand side of equation (4) can be evaluated and put in the form of a complex number, $\alpha + i\beta$. Then, using n+1 even different values of s where n refers to the degree of the Z-transform and n is an odd integer, a set of n+1 equations, such as the following can be formed:

$$
\begin{aligned}
C_o + C_1\gamma_1 + C_2{\gamma_1}^2 + \ldots + C_n{\gamma_1}^n &= \alpha_1 \\
0 + C_1\delta_1 + C_2{\delta_1}^2 + \ldots + C_n{\delta_1}^n &= \beta_1 \\
C_o + C_1\gamma_2 + C_2{\gamma_2}^2 + \ldots + C_n{\gamma_2}^n &= \alpha_2 \\
&\vdots \\
0 + C_1\,\delta_{\frac{n+1}{2}} + \ldots + C_n\,\delta^n_{\frac{n+1}{2}} &= \beta_{\frac{n+1}{2}}
\end{aligned}
\tag{5}
$$

where $Z = \gamma + i\delta$, and γ^n and δ^n represent the real and imaginary parts of Z^n. The subscripts of the γ's, δ's, α's, and β's correspond directly to the different values of s for which they were calculated. If n were an even integer, the last equation in (5) would be:

$$C_o + C_1 \frac{\gamma_{n+2}}{2} + \ldots + C_n \frac{\gamma^n_{n+2}}{2} = \frac{\alpha_{n+2}}{2} \qquad (6)$$

Solving this set of equations for the constants, C_i, will then define the numerator N(Z) as:

$$N(Z) = C_o + C_1 Z + C_2 Z^2 + C_3 Z^3 + \ldots + C_n Z^n \qquad (7)$$

Thus, the Z-transform of the system G(s) is

$$G(Z) = \frac{C_o + C_1 Z + C_2 Z^2 + \ldots + C_n Z^n}{D(Z)} \qquad (8)$$

The root locus can then be obtained for the sampled system either in the s or Z-plane from equation (8).

3.1.1 The Degree of the Numerator One Less Than That of the Denominator

This is a special case where the term $e(0^+)$ in equation (1) is not equal to zero. For this eventuality it is necessary to make a change in equation (2). If one had a transform such as:

$$G(Z) = \frac{\theta_o(Z)}{I(Z)} = \frac{C_o + C_1 Z + \ldots + C_n Z^n}{D_o + D_1 Z + \ldots + D_n Z^n} \qquad (9)$$

its equivalent difference equation (22) would be

$$D_n \theta_o(O) + D_{n-1}\theta_o(-1) + \ldots + D_o \theta(-n) = C_n I(O)$$

$$+ C_{n-1} I(-1) + \ldots + C_o I(-n) \qquad (10)$$

where $\theta_o(-i)$ represent the values of the output function at past sampling instants and the -i in parentheses determines the sampling instant. The term $e(0^+)$ in equation (1) represents a sampled value of the input function at time zero and the only term in equation (10) which could represent this is $C_n I(0)$, therefore C_n must equal $e(0^+)$. A corrected form for equation (4) for the case of the numerator degree being one less than that of the denominator is

$$N(Z) = \frac{D(Z)}{T} \sum_{n=-\infty}^{n=+\infty} G(s + inw_o) + C_n \frac{D(Z)}{2} \tag{11}$$

Thus, a modification to the set of equations (5) is necessary in order to include this extra term. When this term is included and equation (1) is expanded and evaluated in the same manner as equation (4) we get a system of equations in the form:

$$\begin{aligned} C_o + C_1\gamma_1 + \ldots + C_n\gamma_1^n &= \alpha_1 + \frac{C_n D(\gamma_1)}{2} \\ 0 + C_1\delta_1 + \ldots + C_n\delta_1^n &= \beta_1 + \frac{C_n D(\delta_1)}{2} \end{aligned} \tag{12}$$

which, on collecting terms can be put into the form

$$\begin{aligned} C_o + C_1\gamma_1 + \ldots + C_n(\gamma_1^n - \frac{D(\gamma_1)}{2}) &= \alpha_1 \\ 0 + C_1\delta_1 + \ldots + C_n(\delta_1^n - \frac{D(\delta_1)}{2}) &= \beta_1 \end{aligned} \tag{13}$$

where $D(\gamma_1)$ and $D(\delta_1)$ represent the real and imaginary parts of D(Z) with D(Z) calculated for $s = s_1$.

3.2 Z-Transform Inversion

There are several methods for inverting Z-transforms to arrive at the time solution. The Partial Fraction method and the Inversion Formulae method are particularly suited for hand calculation, whereas, the Power Series method is better suited for machine calculation.

The Partial Fraction method involves breaking a Z-transform into its partial fractions and using a Z-transform table to find the corresponding time functions. This method is probably the best to use if the transfer function has a pole with a non-zero imaginary part. The Inversion Formulae is basically another hand method which is most easily applied to transfer functions with simple poles on the real axis. The Power Series method involves a continuous division of the numerator of the Z-transform by its denominator so that a power series in Z^{-1} results. In this method, if the transformed function, E(Z), includes the input signal

$$E(Z) = GI(Z) = \frac{N(Z)}{D(Z)} \tag{14}$$

the coefficients of the power series represent the value of the time function e(nT) at the n^{th} sampling instant. To show

this let:

$$E(Z) = \frac{a_m Z^m + a_{m-1} Z^{m-1} + \ldots + a_1 Z + a_o}{b_n Z^n + b_{n-1} Z^{n-1} + \ldots + b_1 Z + b_o} \qquad (15)$$

then, dividing the numerator by the denominator by long division will yield:

$$E(Z) = AZ^{m-n} + BZ^{m-n-1} + \ldots \qquad (16)$$

where $n \geq m$ for the system to be physically realizable. Now, since the Laplace Transform of a sampled time function can be shown(12) to be:

$$L\left[e^*(t)\right] \equiv E^*(s) = \sum_{n=0}^{\infty} e(nT)e^{-nTs} \qquad (17)$$

and

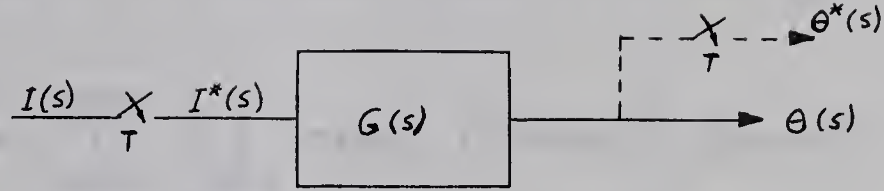
$$Z^n = e^{nTs} \qquad (18)$$

it can clearly be seen that A, B, ... correspond to the respective values of e(nT). If,however, E(Z) does not include the input but can be represented as:

$$E(Z) = G(Z) = \frac{\theta_o(Z)}{I(Z)}$$

then the coefficients cannot represent the value of the time function at the sampling instants but only some ratio of Output to Input. In order to arrive at values for the output time function, one must first obtain the output as some function of the input and the transfer function.

Given the system



Then

$$\theta(s) = G(s)\ I^*(s) \tag{19}$$

In terms of the variable $\theta^*(s)$

$$\theta^*(s) = G^*(s)\ I^*(s) \tag{20}$$

Now following equation (17)

$$G^*(s) = \sum_{n=0}^{\infty} G(nT)\ e^{-nTs} \tag{21}$$

Similarly

$$I^*(s) = \sum_{n=0}^{\infty} I(nT)\ e^{-nTs} \tag{22}$$

and

$$\theta^*(s) = \sum_{n=0}^{\infty} \theta(nT)\ e^{-nTs} \tag{23}$$

Now for a system composed of linear time-invariant elements (25), the response to a unit impulse applied to the system input at $t = 0$ is

$$G^*(t) = \sum_{n=0}^{\infty} G(nT)\ \delta(t-nT) \tag{24}$$

For an impulse modulated signal

$$I^*(t) = \sum_{n=-\infty}^{n=+\infty} I(nT)\ \delta(t-nT) \tag{25}$$

If the system is linear, then, superposition applies and the sampled response to an impulse train is given by

$$\theta^*_o(t) = \sum_{n=-\infty}^{n=+\infty} \left[\sum_{m=0}^{\infty} G(nT)\ I(nT-mT) \right] \delta(t-nT) \qquad (26)$$

The output sample sequence which is of interest is represented by the bracketed term of equation (26) so that

$$\theta(nT) = \sum_{m=0}^{\infty} G(mT)\ I(nT-mT) \qquad (27)$$

or

$$\theta(nT) = \sum_{m=0}^{\infty} G(mT)\ \delta(t-mT)\ I(nT) \qquad (28)$$

Now transforming equation (20) yields

$$\theta(Z) = G(Z)\ I(Z) \qquad (29)$$

Now G(Z) can be represented as an infinite series of the form

$$G(Z) = A_1 Z^{-1} + A_2 Z^{-2} + A_3 Z^{-3} + \ldots \qquad (30)$$

From equation (30) it can be seen that G(mT) in equation (28) can be replaced by A_m for $0 < n \leq \infty$ thus:

$$\theta(nT) = \sum_{m=0}^{\infty} A_m\ \delta(t-mT)\ I(nT) \qquad (31)$$

where A_m are the coefficients of G(Z).

This shows then, that when using the transfer function to arrive at a value of the time function $\theta(nT)$, it is necessary to sum the product of A_m and $I(nT)$ for m going from zero to n instead of simply the coefficients of the Z-transform being the values of the time function at that sampling instant.

4. THE ROOT LOCUS

The control systems analysis program does the calculations necessary to establish the root locus. The control system is specified by its feed-forward and feedback loop transfer functions. Points on the root locus, defined by $G(s)/s{=}a{+}ib = -1/K$, are found by a systematic grid search and interpolation procedure over a user specified area in the s-plane.

The Root Locus Technique is based on satisfying the characteristic equation of the system and any point satisfying this equation is a point on the root locus. The characteristic equation can be expressed as:

$$1 + K\,G(s) = 0 \tag{32}$$

where $G(s)$ is the open loop transfer function of the system, and K is the loop gain. Rearranging we arrive at

$$G(s) = -\frac{1}{K} \tag{33}$$

Now for various values of s, $s = \delta + iw$, values for $G(s)$, $G(s) = q = u + iv$, can be calculated. Since $-1/K$ has to be a real number, v must equal zero for equation (33) to be satisfied. The procedure, then, is to calculate values of $G(s)$ for various values of s along a line of constant δ or w in the complex plane and note the sign of v. When a change in the sign of v occurs, it signifies that the root locus has

been crossed. The point in the s-plane which satisfies equation (33) can then be located more accurately by a search between grid points. The loop gain corresponding to this value of s is then calculated from

$$K = -\frac{1}{u} \tag{34}$$

This type of procedure is carried out over the whole specified scan area.

The method is similar to the principle incorporated in Evan's Spirule, but is suited to use by a high-speed digital computer. Once the root locus has been determined as a list of points, from which a graph may be obtained, the results can be examined to determine system stability, (A.9.2).

The criterion which is used in this type of analysis is that for a given value of loop gain K, all the roots of the characteristic equation must exist in the left half of the s-plane. This guarantees that all the roots have negative real parts and thus the time solution in response to a bounded disturbance will not become unbounded for increasing time. This is defined as a stable system.

The values of K, for which the real part of each root of the characteristic equation is less than zero, can be determined from either the complex Z or s-plane for a sampled-data system. Generally, the Z-plane root locus is almost exclusively used for analysis for sampled-data systems

and the s-plane root locus for continuous systems. The Control Systems Analysis program, however, has made the s-plane available for the stability analysis of both types of system, thus, the user does not have to be familiar with both. To arrive at the s-plane root locus for a sampled-data system the following operation sequence is performed.

a) The Z-transform is calculated.

b) The first scan point in "s" is determined according to user specification.

c) This scan point is converted to its equivalent in Z.

d) A system number in the Z-plane is calculated.

e) This system number is examined for its proximity to a root locus.

f) If it is on the root locus, the point is printed out along with its value in the s-plane. If it is not, the next scan point is selected in s and the process repeated from b).

There are some undesirable characteristics to this procedure; the scan area in Z is determined by that specified in the s-plane. This, in some cases, can by very undesirable. Points in the Z-plane are determined from the s-plane by the mapping function $Z = e^{sT}$. By using scan points determined in the s-plane, it can be seen that the area in the Z-plane over which the scan takes place is severly limited. For example, using this program Z can be evaluated only in the

first quadrant. In order to reach zero on the Z-plane, s would have to be given the value of minus infinity. This severely limits analysis of a system in the Z-plane.

It has been mentioned that the root locus of a sampled-data system can be converted to the s-plane. It might be thought that this should be enough for a good stability analysis, however, analysis of a sampled system in the s-plane does hide certain influences, particularly the full influence of the sampling interval T.

It was noted in the transient response calculations for continuous systems, that very small sampling intervals caused multiple poles at Z = 1. The occurrence of which led to system instability. This is obvious since the $\lim_{T \to 0} (e^{sT}) = 1$. This instability problem does not usually occur in physical systems because of the large sampling intervals used. The s-plane shows the instability of the system in this case, but it is hard to find the reason until the root locus of the system is also examined in the Z-plane. The reason for the instability problem then becomes obvious.

In order to avoid the problems given above, a program change was made which now provides a Z-plane root locus determination over a scan area set in the Z-plane. This is an optional calculation and requires a flag set by the user. With this flag set only the Z-plane root locus is determined.

5. THE CONTROL SYSTEMS ANALYSIS PROGRAM

5.1 The Original Program

This program was originally capable of calculating a root locus for a linear feedback system, with or without time delay, and also capable of calculating a Z-transform of a transfer function which was specified in Laplace transform notation. Using the Z-transform conversion ability it chould then provide the root locus in the s-plane for sampled-data systems.

In order for the program to function, it was necessary to arrange the block diagram of the system to be analyzed into forward and feedback loops. When this was accomplished, the transfer functions were entered with those in the feedback loops entered first and those of the forward loops last. The program would then evaluate and combine these transfer functions to arrive at an overall system number for that value of s considered.

This aspect of the program has a limitation in that the components of the block diagram must be manipulated by the user in order to achieve the necessary input form. As this limitation was not insurmountable, only an enquiry into the programming necessary to provide this function for the user was made. Various programming languages could be used in this capacity, however, no attempt was made to

implement this, as the main concern was the calculating ability of the program, and the types of systems and problems for which the program could be used.

The original program consisted of 15 subroutines and the mainline program DK1. These subroutines and their relation to each other are shown in Figure (2).

Each subroutine has one specific purpose or step in the overall calculation to perform and can be called a number of times by other subroutines to perform its calculation. A list of these subroutines with their purposes is provided in Table (1).

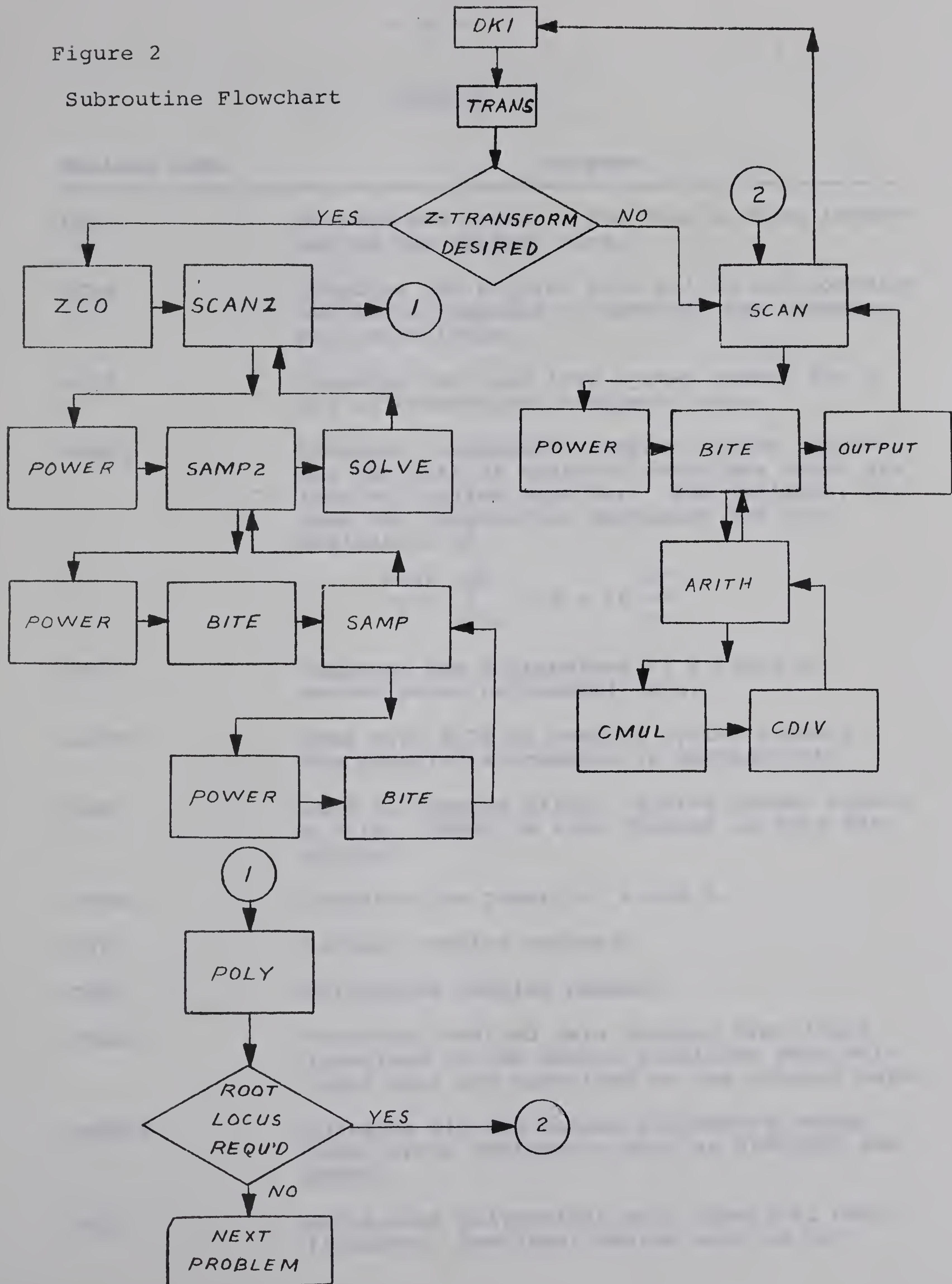
5.2 <u>The Present State of the Program</u>

During the investigation of the C.S.A. program it was seen that the Bode and Nyquist stability analysis plots could be easily incorporated. This was done and the necessary subroutines tested using the example problem from Schilling(18).

A method was also established whereby the transient response of certain systems could be obtained using the Z-transform capability of the program as a basis. The method by which this is done was outlined previously in Section 3.2. This method depends on the program's ability to calculate Z-transforms for transfer functions in s. The Z-transform program requires that the denominator of the s-transfer function be factored into first and second order terms. A Share Library

Figure 2

Subroutine Flowchart



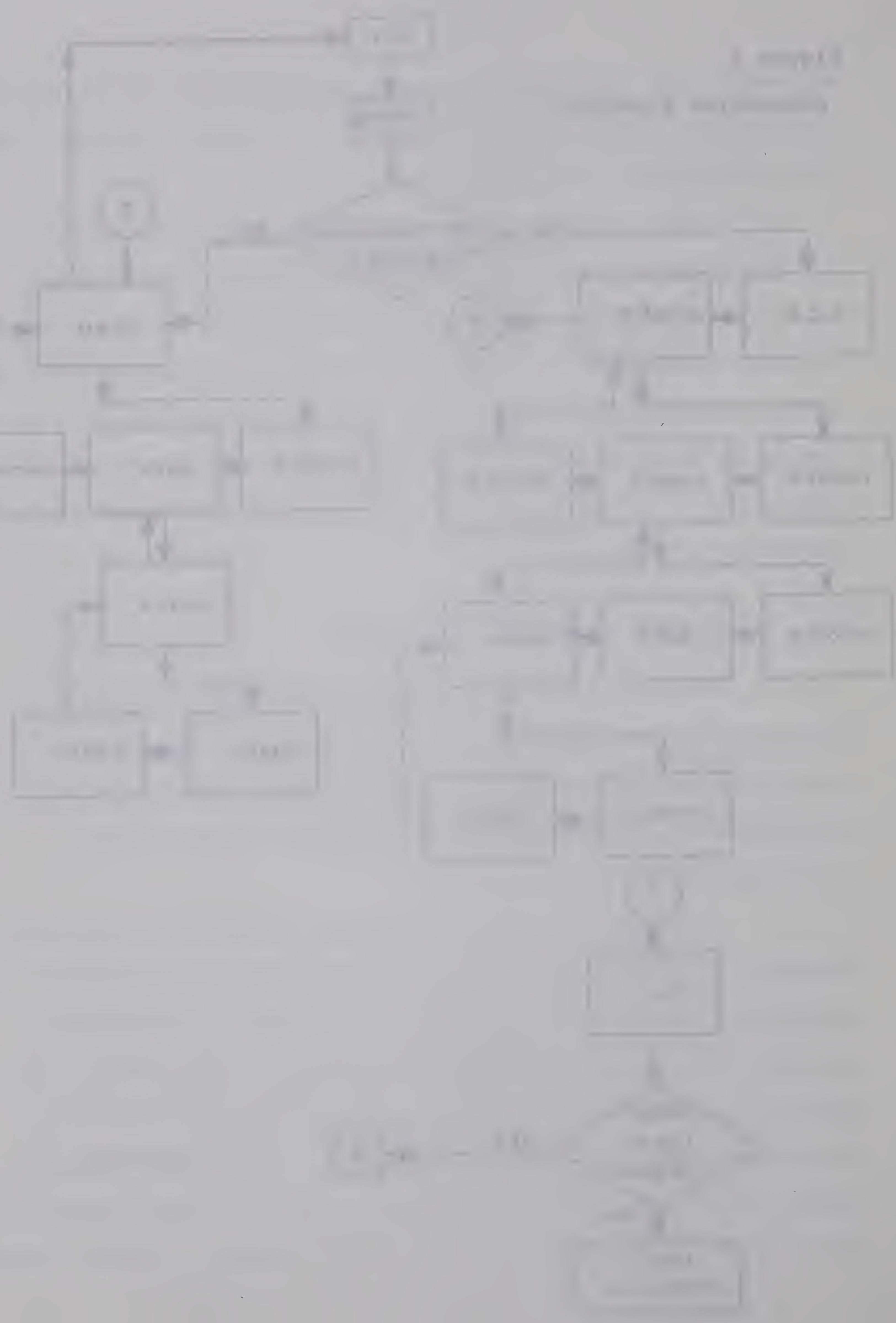


TABLE 1

Routine Name	Purpose
DK1	To read and organize the flow of data according to the control card.
SCAN	Computes the s-plane scan points and contains the logic required to identify the crossing of a root locus.
BITE	Computes the open loop system number for a set of forward and feedback loops.
SAMP2	Computes independent complex system numbers for two sets of transfer functions which are then multiplied together. For instance, it does the computation necessary for the evaluation of $\frac{D(Z)}{T} \sum_{n=-a}^{n=a} G(s + in \frac{2\pi}{T})$.
ZCO	Computes the Z-transform of a first or second order polynomial in s.
ARITH	Used with BITE to compute system numbers. The required arithmetic is carried out.
SAMP	Used to compute $G^*(s)$. System number equals $\alpha + i\beta$. ERROR is also checked in this subroutine.
POWER	Computes the powers of s and Z.
CDIV	Divides complex numbers.
CMUL	Multiplies complex numbers.
TRANS	Transfers desired data changes from input locations to the proper locations when multiple runs are specified on the control card.
OUTPUT	Contains all the output statements except those error statements such as SINGULAR and ERROR.
POLY	Multiplies polynomials with numerical coefficients. Resultant degree must be ≤ 35.

TABLE 1 (continued)

Routine Name	Purpose
SCANZ	Determines the set of equations to be solved to get the numerator coefficients.
SOLVE	Solves the algebraic equations set up in SCANZ by the Gauss Elimination Method. Will print "singular" if the determinant is singular.

Program(23) was used to calculate the roots for a polynomial greater than second order and these roots were then used as input data for the C.S.A. program. The Share program is not incorporated into the original Control Systems Analysis program, however, this could be done.

A modification was made to the Control Systems Analysis program so that a Root Locus for a sampled-data system could be calculated and plotted in the Z-plane. The modification consists of another calculation path in which the Z-plane is scanned in the same manner as the s-plane when finding a root locus for a continuous system. In order to implement this, a flag is set by the user and the scan area in the Z-plane, over which the user is interested, is set in the same manner as that in the s-plane for a continuous system.

Provision was made for the plotting of these calculated Root Locus points using the University of Alberta Computing System's "Autoplot" routine.

The more complete Z-plane root locus has been found particularly useful in the design of sampled-data or digital controllers for given systems. This aspect will be treated later.

The new calculation path designed to give the Z-plane root locus was checked for proper functioning by the comparison of the digital solution and that presented in an article by Slaughter(19) for the same problem.

Keeping the user of the program in mind, it was thought that a more explanatory form of Input Data printout should be available. Consequently another output routine, "DPRINT", was written which prints out the control card specifications along with their explanation. This allows the user to make an immediate check of the input data without having to refer back to the literature.

A main step in the calculation of the transient response for, or the study of, a system controlled with a sample-data controller was the formation of the closed loop transfer function (Section 7.4). The formation of this transfer function required two of the three runs usually necessary for the solution, in addition to the hand multiplication and addition of polynomials with the consequent loss in accuracy. The program has been modified so that only two runs of the program are necessary and all the multiplication and addition of polynomials is done by the machine.

Figure (3) and (4) constitute a flow diagram showing the calculation paths of the present modified program. The added calculation paths and blocks are indicated by dashed lines.

Figure 3 - Calculation Flow Diagram of Modified C.S.A. Program

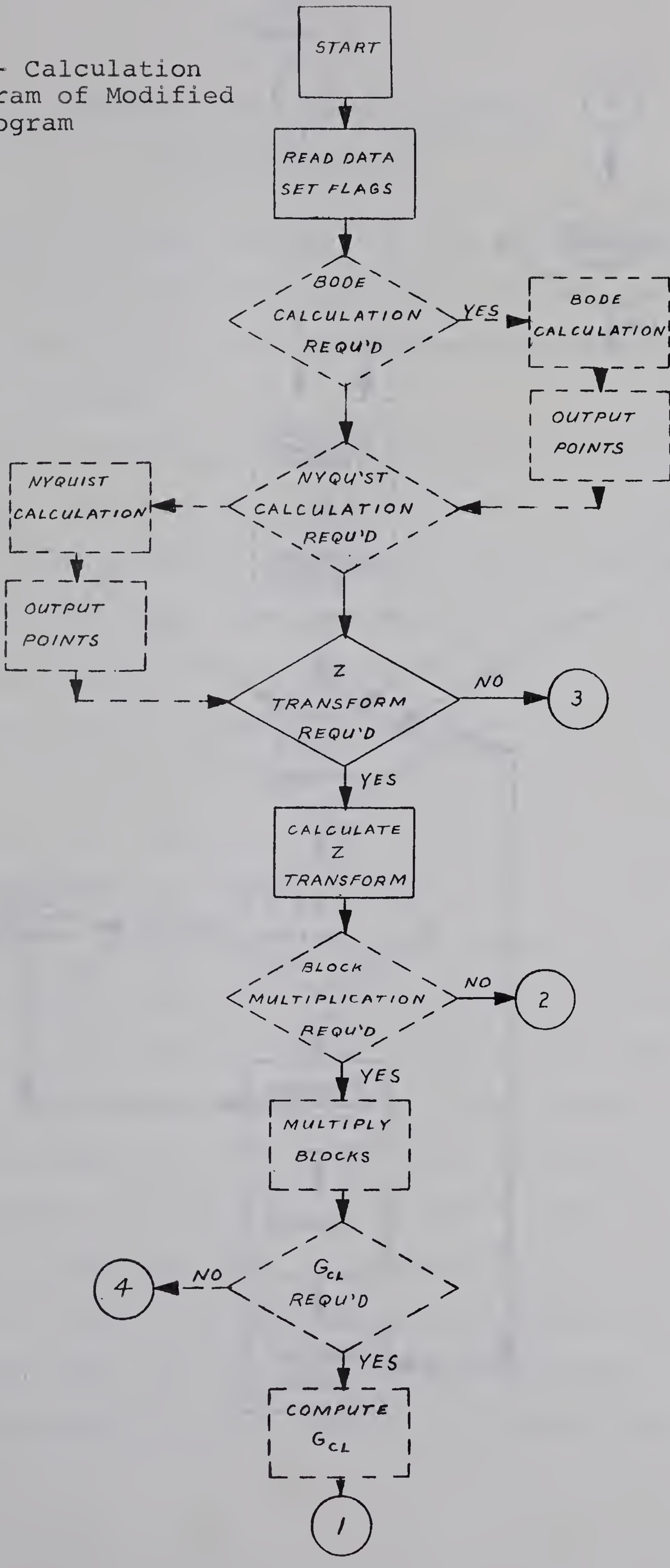
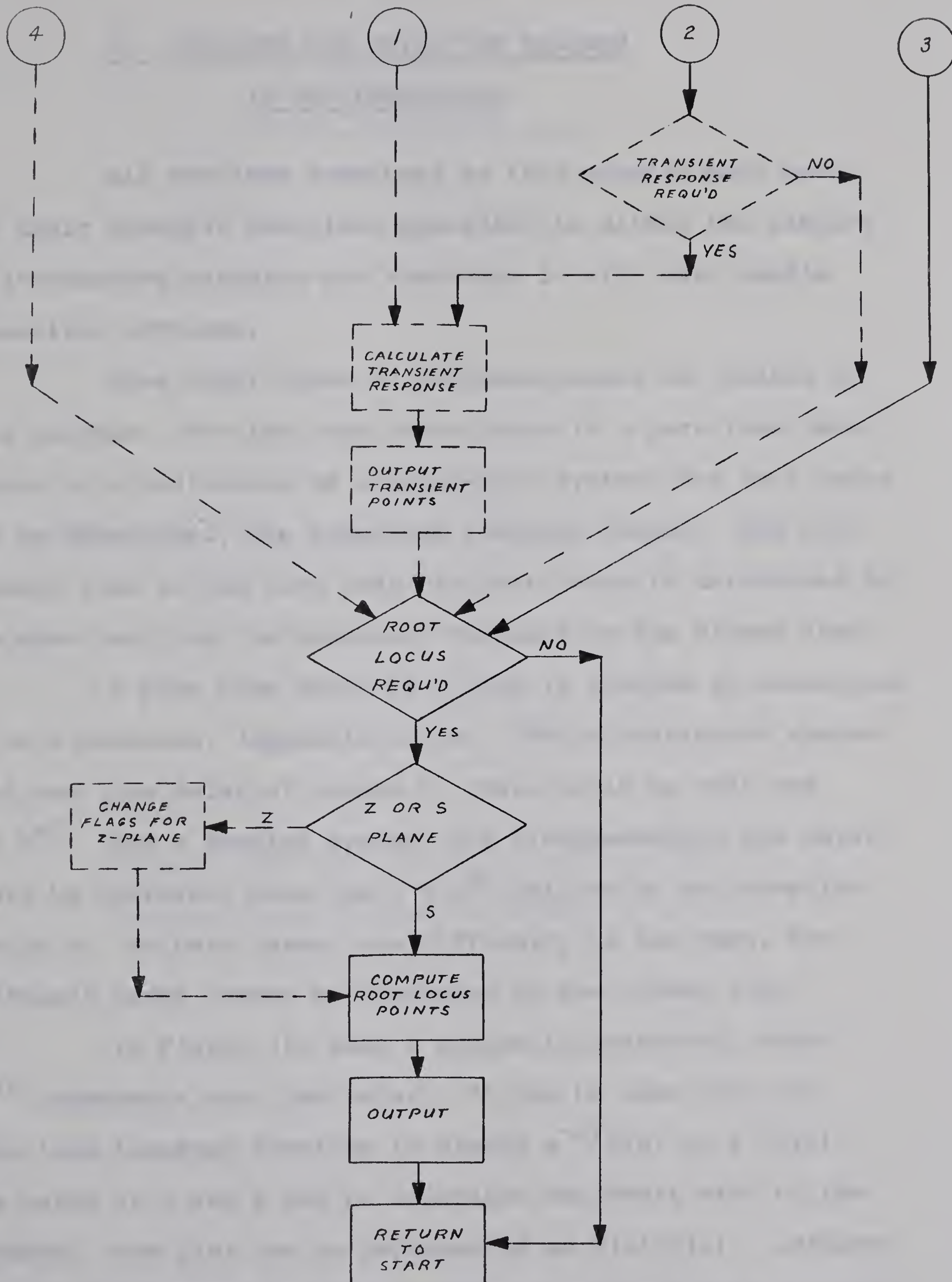


Figure 4


4
1
2
3
TRANSIENT RESPONSE REQU'D
NO
YES
CALCULATE TRANSIENT RESPONSE
OUTPUT TRANSIENT POINTS
ROOT LOCUS REQU'D
NO
YES
CHANGE FLAGS FOR Z-PLANE
Z
Z OR S PLANE
S
COMPUTE ROOT LOCUS POINTS
OUTPUT
RETURN TO START

6. PROBLEMS FOR WHICH THE PROGRAM IS NOT APPLICABLE

All problems submitted to this program must have all their transfer functions specified in either the Laplace or Z-transform notation and therefore it will only handle linearized problems.

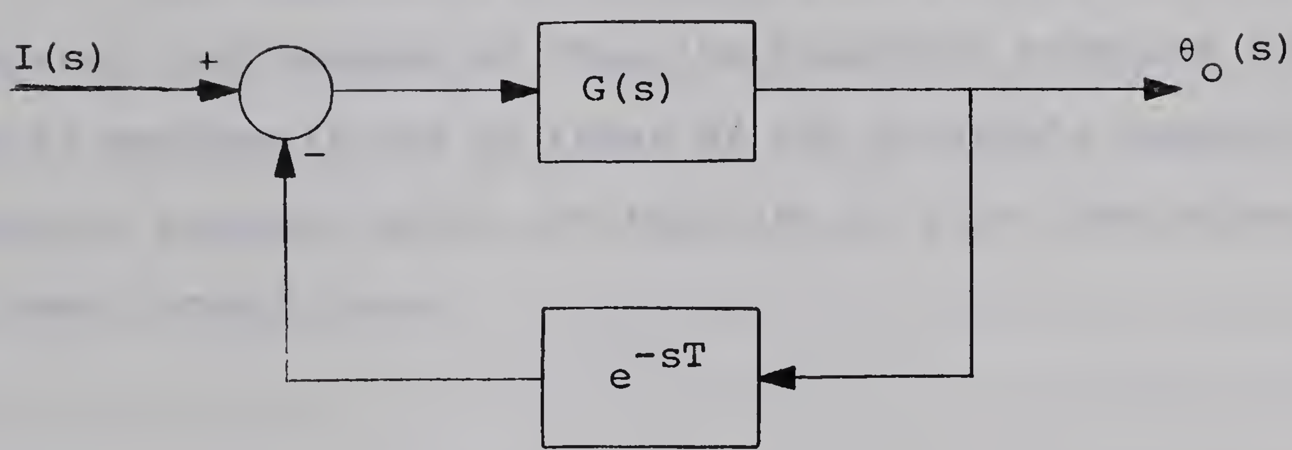
Some other types of problems cannot be treated by this program. For the case where there is a pure time delay either in a continuous or sampled-data system, the root locus can be determined, the transient response cannot. The difference lies in the fact that the root locus is determined by the open loop and the transient response by the closed loop.

A pure time delay in a loop is treated by converting it to Z-notation, (Appendix A.9.2). For a continuous system with one time delay of length T, there would be only one $Z = e^{sT}$. For a sampled system, the Z representing the delay would be different than the $Z = e^{sT}$ defined by the sampling period T. In both cases, the difficulty is the same, the different terms cannot be separated in the closed loop.

In Figure (5) such a system is presented, where e^{-sT} represents the time delay. It can be seen that the open loop transfer function is simply $e^{-sT}G(s)$ or $Z^{-1}G(s)$. The terms in Z and s can be separated and dealt with by the program. Now G(s) can be represented as N(s)/D(s). Letting G(s) be represented in this fashion, the closed loop transfer

Figure 5

Control Loop With Time Delay



Forward Loop Transfer Function = G(s)

Feedback Loop Transfer Function = e^{-sT}

function of the system becomes:

$$G_{cL}(s) = \frac{\frac{N(s)}{D(s)}}{1 + e^{-sT}\frac{N(s)}{D(s)}} \tag{35}$$

or

$$G_{cL}(s) = \frac{N(s)}{D(s) + Z^{-1}N(s)} \tag{36}$$

For equation (36) the terms in Z and s cannot be separated, and because of this the transient response for this type of problem is out of range of the program's capabilities. A similar analysis holds for the case of pure time delay in the feed forward loop.

7. APPLICATIONS

The main object of this research was to find the types of chemical engineering control problems the program could be applied to and to investigate a possible widening of its applications.

There are two main classes of control systems, linear and non-linear; non-linear being the majority, and also the more difficult to treat. Each main class may be divided into two subclasses, one is the continuous and the second is the discrete or sampled system. Except for one case, the program is unable to cope with the non-linear type systems, however, it deals very well with most of the linear types. The following will be a summary of the problems encountered in treating each type of problem using the Control Systems Analysis program and an outline of the steps used to carry the problem to solution.

7.1 Root Locus for Continuous Linear Systems with or Without Pure Time Delay

The simplest control problem is the single feedback loop, linear, continuous system. No problems were experienced when analyzing this type, however, an outline of the data which must be provided about the system is presented so that the exceptions can be noted in following types of problems. To enter this type of problem for stability analysis, a minimum number

of specifications must be made. There must be specified:

a) the number of feedback and forward loops;

b) the order of each polynomial in the transfer function and whether it is in Z or s notation;

c) the scan area and increment on both axis; and

d) the number of different solutions wanted for the problem. Parameter changes may be made for the same problem.

With the above information a root locus calculation can be performed.

The next type of system is one in which there is a pure time delay in either the forward loop, or in the feedback loops. In treating this problem, the Z-transform capability of the program is used. The factor e^{-sT} representing the pure time delay is entered into the transfer function as Z^{-1}, since $Z = e^{sT}$. No Z-transform calculation of the s-transfer function is specified by the user, however, the time delay is entered as data along with the highest power of Z. This, of course, is equal to one. An example problem, Appendix (A.9.2), showing the effect of various pure time delays on the stability of a control system is shown along with the input information necessary for the correct designation of the problem.

For the above two types of problems only the root locus was mentioned as the stability criterion, however, because of additions made to the program, Nyquist plots and Bode plots are also available. These are produced when and only

when specifically asked for by the user. This involves setting the span of, and the increment of, the frequency over which the user wishes to examine the system. The user must also set a flag in order to direct the program to do either or both of the calculations necessary to produce the two plots. An example problem has been included, Appendix (A.9.3), showing the computed graphs for these three types of analyses.

7.2 The Study of Sampling Interval

The insertion of a sample and hold device into a loop has a destabilizing effect on a control system. The degree to which this effect is felt by the system depends a great deal on the sampling frequency used. Generally, the smaller the sampling interval, the less the destabilizing effect the sampler has on the system. It would be advantageous to a systems engineer to be able to determine beforehand what effect sampling intervals would have on a given system, and to be able to determine where the limit of stability for the system occurs. It would then be possible to specify the sampling rate, and design on this basis with some assurance of overall system stability.

The program is well suited to this type of study. Using this program, a system can usually be studied through a root locus analysis to determine for what gains instability occurs for various sampling periods. Study using the transient

response depends on the placing of the sample and hold. A different method than the one outlined in Section 3.2 may have to be used if the sampler is not in the forward loop. In this way the program could be used as a valuable tool in the conversion of a continuous control system to a sampled system in preparation for digital control.

7.3 Transient Response Calculation

7.3.1 Continuous Systems: Change in Set-Point

With the addition of another subroutine "PDIV", Appendix (A.5), it was found that the Control Systems Analysis program could be used in the calculation of the transient response of some systems. The calculation for continuous systems involves obtaining the Z-transform of the closed loop s-domain transfer function and inverting this Z-transform. The inversion results in values for the weighting sequence of the output. The mathematical formulation is outlined in Section 3.2.

A continuous system may be represented as shown in Figure 6-a, for which the open loop transfer function is:

$$G_{oL}(s) = G_1(s)\ G_2(s) \tag{37}$$

and the closed loop transfer function is:

$$G_{cL}(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)} \tag{38}$$

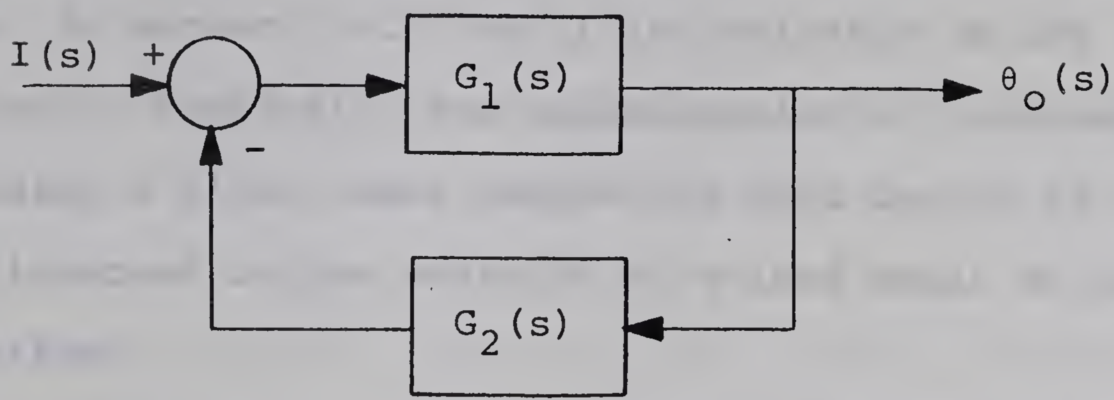
Using the closed loop transfer function the block diagram shown in Figure (6-a) can be converted to that shown in Figure (6-b). By inserting a sample and hold before the block representing the system, the continuous system can now be transformed to a sampled system to which Z-transform theory can be applied, Figure (6-c).

As would be expected, the sampling period, T, chosen for the sampler which converts the continuous signal to the closed loop system to a sampled signal has a great effect on the overall response of the system. Now, for the sampled system to exhibit the same type of transient response as the continuous system the sampler should have little effect on the system. Since it is the roots of the system which determine the type and manner of response, a measure of the effect of the sampler on the system is the difference between the root loci of the sampled closed loop system and the root loci of the continuous closed loop system. When the root loci of the sampled and continuous systems agree as to the position of their poles and zeros on the s-plane, the trajectories of the roots in the complex area of the s-plane, as well as in the values of the relative gains along the locus, the inverse of the Z-transform multiplied by the value of the input signal at that time will be the value of the output function in the time domain. The inverse of the Z-transform is obtained by the power series method.

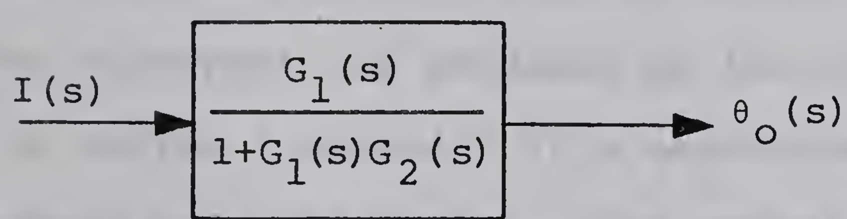
Figure 6

Block Diagram for Continuous System
Transient Response Calculation

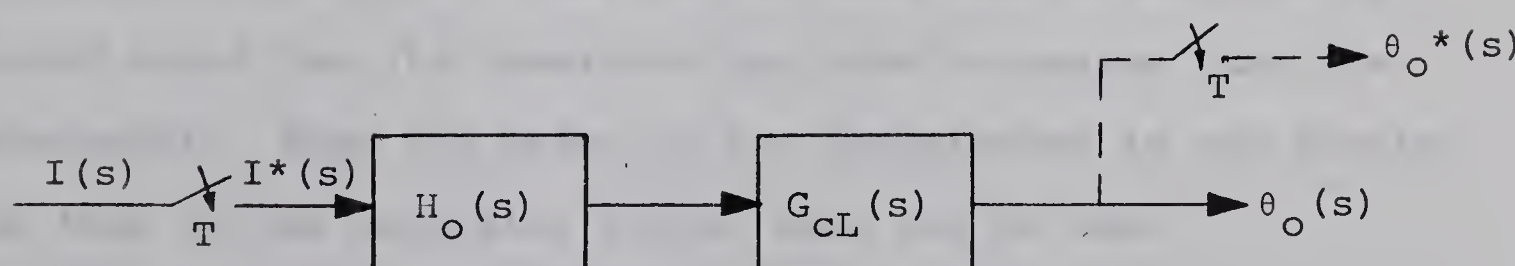
(a)

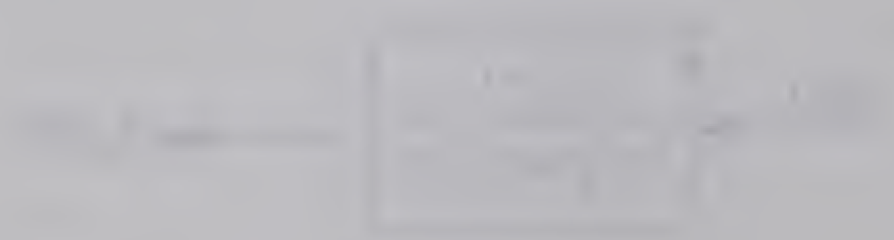


(b)



(c)





When using a sample and hold device there is a certain amount of lag associated with it. For instance, in most cases a zero-order hold will give a lag of half a sampling period. To correct this lag it is desirable to add a fictitious lead. Similarly, the approximation of continuous functions using a first order sample and hold device is, in most cases, improved by the addition of a lead equal to one sampling period.

If the transfer function of the closed loop turns out to have the degree of the denominator only one greater than that of the numerator, a first order hold is necessary. This eliminates the need for the user to define a new Z to account for the transport lag produced by the hold. It would be necessary to define a second Z if a zero-order hold were used as the sample and hold device. The use of the first order hold is made necessary because the program lacks the capability to account for a different Z, other than the one being used for the transform, and at the same time calculating a special Z-transform. This is the case when a function is treated which has its numerator one less in degree than its denominator. When the order of the denominator is two greater than that of the numerator either hold can be used.

An example of each type of system is presented in the Appendix, (A.9.4.1, A.9.4.2).

7.3.2 Continuous Systems: Change of Load

All the systems treated previously were disturbed by a step change in the set-point of the controller. In chemical systems, disturbances also occur because of changes in the load variable. Although the root locus analysis is the same for both types of disturbance, since it deals with only the characteristic equation, the overall response is different. This can be attributed to the change in the numerator of the transfer function of the system.

A block diagram with the load variable included is shown in Figure (7-a). The equation representing this system is

$$\theta_o(s) = \frac{G_p(s)}{1+G_m(s)G_p(s)}\,\theta_i(s) + \frac{G_L(s)}{1+G_m(s)G_p(s)}\,\theta_L(s) \qquad (39)$$

and when studying the effect of load change, $\theta_i(s)$ can be set to zero and for $G_m(s)G_p(s) = G_{oL}(s)$:

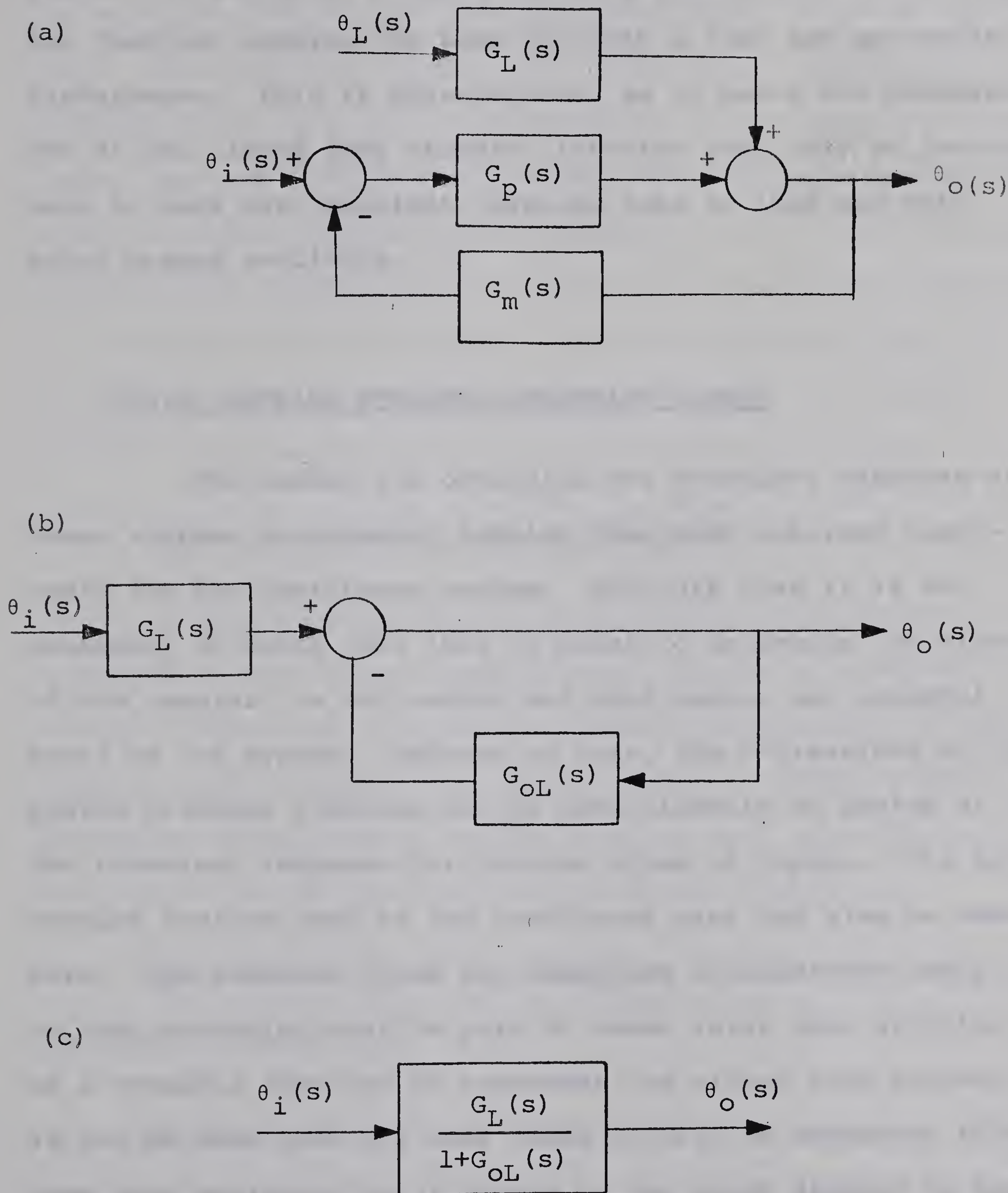
$$\theta_o(s) = \frac{G_L(s)}{1 + G_{oL}(s)}\,\theta_L(s) \qquad (40)$$

A block diagram representing equation (40) is shown in Figure (7-b) which can be condensed to that of Figure (7-c).

Thus, to arrive at the response to a step input or any other disturbance in the load variable, it is only necessary to follow the steps previously outlined for the response of a continuous system to a change in the set-point

Figure 7

Block Diagram Sequence for the Transient Response for a Continuous System (Load Change)



point. It should be noticed that the denominator of the transfer function remains the same for both a load and set-point disturbance. This is advantageous, as it means the denominator of the closed loop transfer function must only be factored once to make the transient response both to load and set-point change available.

7.3.3 Sampled Systems: Set-Point Change

The method for obtaining the transient response of these systems is somewhat simpler than that outlined previously for the continuous system. For this case it is not necessary to match root loci in order to determine the effect of the sampler, as the sample and hold device are integral parts of the system. Because of this, the Z-transform of the system transfer function can be used directly to arrive at the transient response for various types of inputs. The inversion routine used in the continuous case can also be used here. The standard rules for combining Z-transforms apply so that attention must be paid to these rules when arriving at a transfer function to represent the closed loop system. It can be seen that for some cases it will be necessary to do some hand manipulation of blocks in the block diagram as the program does not have block manipulation capability.

In the procedure for obtaining the transient response

or root locus for an error-sampled system, it was necessary to multiply, and/or add different polynomials together by hand. This was both tedious and inaccurate, consequently an addition (A.3, A.4, A.6, A.7) was made to the program to relieve the user of this calculation. The addition can be used only for unity feedback error-sampled systems. The main function of this addition is to relieve the user of the necessity of combining the Z-transform of the controller and that of the process and to reduce the number of runs. An example problem is presented (A.9.6) to illustrate the method.

7.3.4 Sampled Systems: Load Change

The Calculation of the transient response of a system to a load change differs from that for a set-point change. Figure (8) is a block diagram representing an error-sampled control system with unity feedback. Given a load change and taking the set point I(s) as zero, the equation for the output can be obtained from the block diagram as:

$$\theta_o^*(s) = \theta_L G_L^*(s) - D^*(s) G_p^*(s)\ \theta_o^*(s) \tag{41}$$

Taking the Z-transform

$$\theta_o(Z) = \theta_L G_L(Z) - D(Z) G_p(Z)\ \theta_o(Z) \tag{42}$$

from which the closed loop transfer function

$$\theta_{O_{CL}}(Z) = \frac{\theta_L G_L(Z)}{1 + D(Z) G_p(Z)} \qquad (43)$$

can be formed. $\theta_L G_L(Z)$ represents the $Z[\theta_L(s) G_L(s)]$.

It can be seen that the input function is not a separate identity in the numerator, thus, it must be Laplace transformable. Because of this the calculation of points on the transient response curve is also different. These points will now be the coefficients of the power series in Z^{-1}, Section (3.2), when the power series inversion method is applied, this was not the case for a set-point change. Thus, the transient response calculation for a sampled-data system with a load change requires a variation in the procedure normally used for the transient response to a set-point change. The transfer function $\theta_L G_L(Z)$, the numerator of equation (43), must be calculated from $\theta_L(s) G_L(s)$ before the overall transfer function, equation (43) can be obtained.

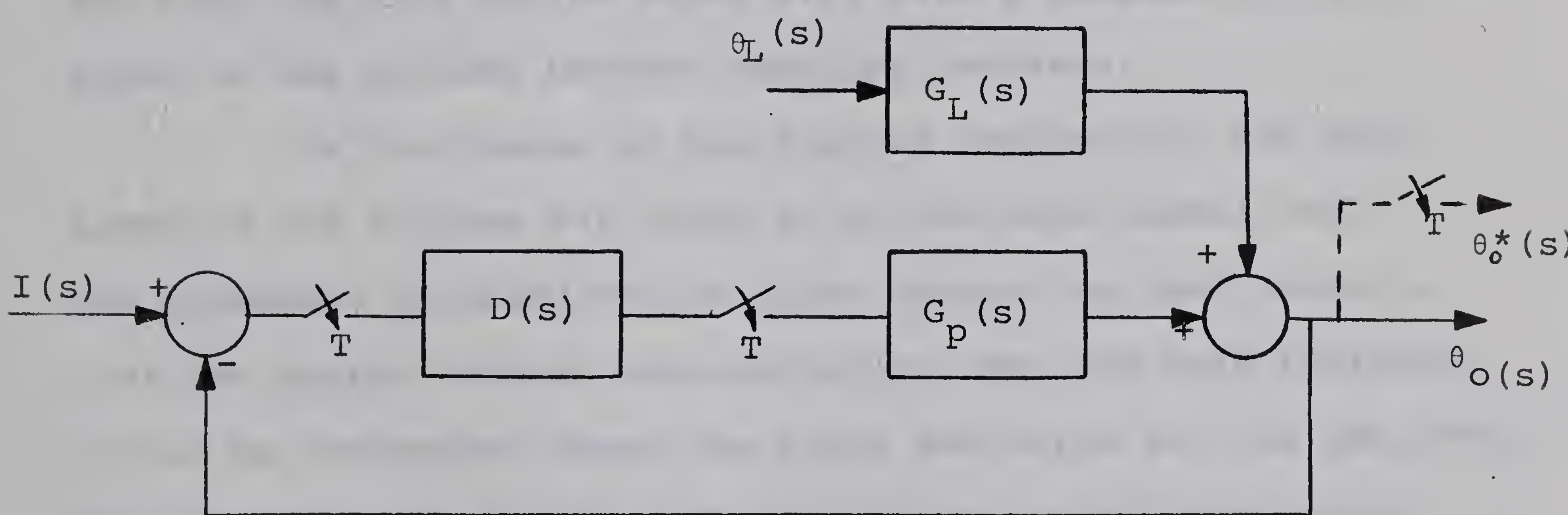
7.4 <u>The Specification and Design of Digital Controllers</u>

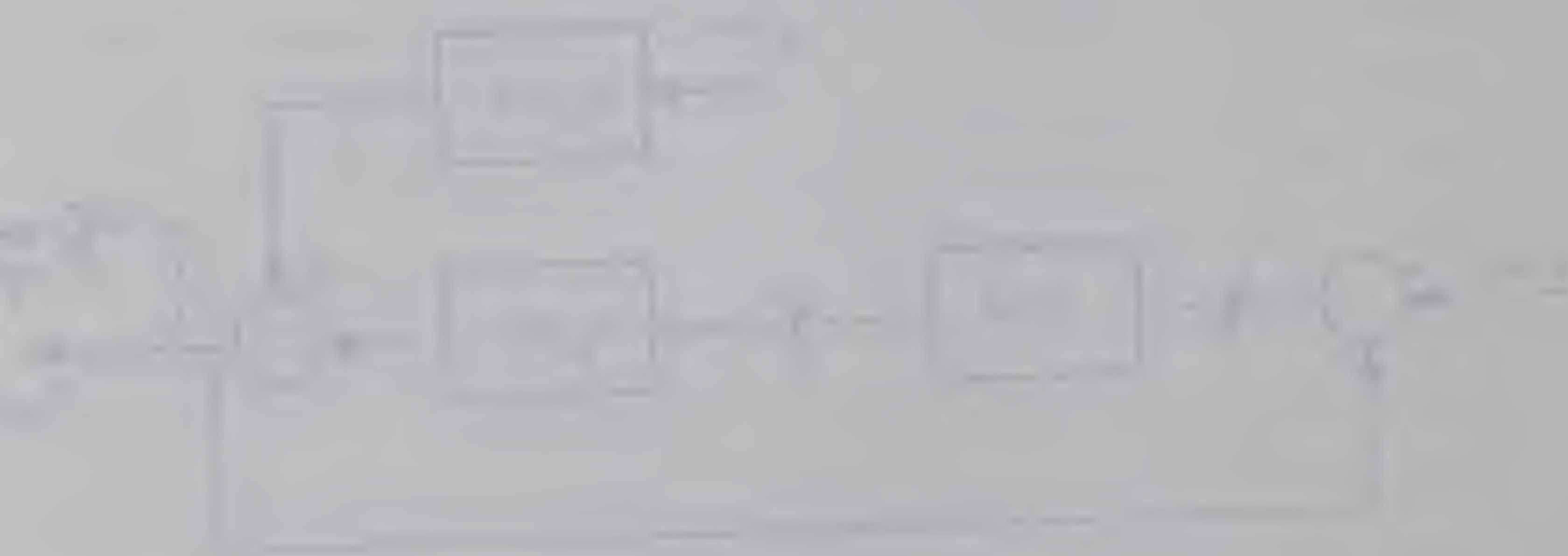
<u>7.4.1 Designing a Digital Controller for a System</u>

An investigation into the application of the Control Systems Analysis program to the specification and design of digital controllers was made. The particular application is in the determination of a digital controller to control a system previously controlled by a continuous controller.

Figure 8

Block Diagram for Load Change





A continuous system is represented by the block diagram in Figure (9-a), where $G_c(s)$ is the transfer function of the controller and $G_p(s)$ is the transfer function of the process.

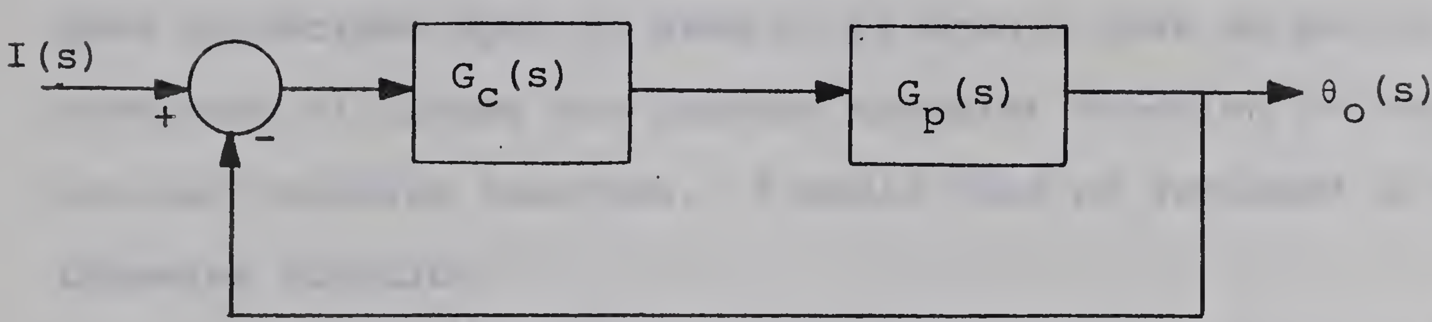
Now, in converting to a digital controller, which could be a digital computer, modifications are made to Figure (9-a) to represent the digitally controlled system. This is shown in Figure (9-b). $D_c(Z)$ represents the digital controller and H(s) the hold device which will give a constant analogue input to the process between sampling instants.

In the design of the digital controller, the root locus in the Z-plane was found to be the most useful tool. The procedure followed was to first obtain the root locus in Z of the system without the controller, but the hold included. It can be determined where the poles and zeros in Z of the controller should be placed from an examination of this locus. This step constitutes one run of the program. With the controller poles and zeros decided upon a new root locus is calculated either in s or Z, depending on the user's preference, (Appendix A.9.6). The advantage of one type of root locus versus the other varies with the problem. The object of this step is to obtain a knowledge of the overall loop gain for which the system will be stable. In some cases one root locus is better than the other for this. The user now picks the overall loop gain for which he wants to examine the transient

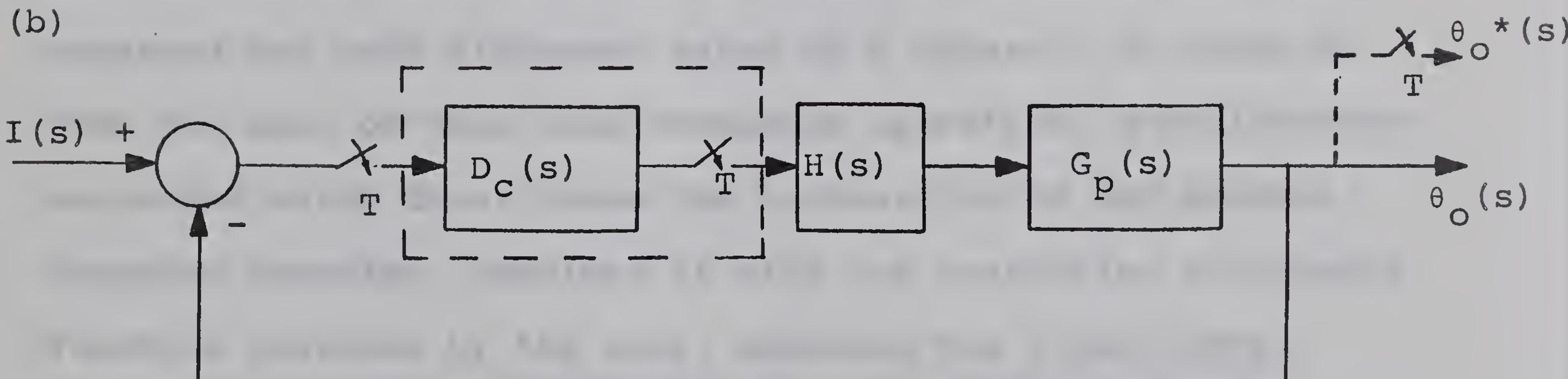
Figure 9

Block Diagram Sequence for Digital Controller Implementation

(a)



(b)



response. This is the end of the second run. The third run is to obtain the transient response of the system for the loop gain decided upon previously in the second step. The program always calculates the transient response of the system for the loop gain K equal to one. This means that when an overall loop gain is decided upon in step 2, it should then be put in the numerator of either the process transfer function or the controller transfer function. K would then be included in the transfer function.

Before the modification mentioned in Section 7.3.3, the closed loop Z-transfer function with the value of gain inserted in the numerator was calculated by the user. This involved both multiplication and addition of polynomials and was necessarily done with the Z-transfer function, and was repeated for each different value of K chosen. In order to free the user of this time consuming operation, a modification was added which first takes the Z-transform of the process transfer function, combines it with the controller Z-transfer function provided by the user, computes the closed loop transfer function and from this calculates the transient response. To use this modification the flag LOOP(Appendix A.5) must be set equal to one. The sample period, total time, units of time and the size of the step in the setpoint must also be set (Appendix A.5). The order of each term, the number of terms, and the coefficients of these terms making up the controller transfer function in Z, must be read in (Appendix A.4).

An example calculation is presented in Appendix (A.9.6), with the accompanying input data.

When using a process control computer for control, the controller for the process is in the form of a computer program, so that once the transfer function in Z has been determined, it must be programmed for computer use. An outline of the method to do this is presented in Kuo(12). It is sufficient to say that once a Z-transfer function has been determined for the controller, a digital program can be built directly to realize this transfer function.

It should be noticed that once the root locus for the open loop is determined, a loop gain is picked from it for which the transient response of the system is calculated. Various gains can be picked from this root locus of the system to give different transient responses. Thus, for each transient response calculation it is not necessary to determine a root locus. This results in a considerable saving in calculation time.

7.4.2 Direct Replacement of a Continuous Controller by a Digital Controller

The program can be used not only in specifying a digital controller for a system, but also specifying a digital controller to provide the same response as when the system was under continuous control.

For a continuous system, Figure (9-a), the open loop transfer function can be represented as:

$$G_{oL}(s) = G_{cI}(s)G_p(s) \tag{44}$$

The Z-transform of which is

$$G_{oL}(Z) = G_{cI}G_p(Z) \tag{45}$$

For an error-sampled system, Figure (9-b), the open loop transfer function is

$$G_{oL}(z) = D_c(Z)H\ G_p(Z) \tag{46}$$

Now for the response of the two systems to be the same, the root loci must be the same, or,

$$1 + KG_{cI}G_p(Z) = 1 + KD_c(Z)HG_p(Z) \tag{47}$$

from which

$$D_c(Z) = \frac{G_{cI}G_p(Z)}{H\ G_p(Z)} \tag{48}$$

A transfer function for $D_c(Z)$ is to be determined. Dividing the terms in the right hand side of equation (48) into their respective numerator and denominator terms, equation (48) can then be written as:

$$D_c(Z) = \frac{\dfrac{N_{cI} N_p(Z)}{D_{cI}(Z) D_p(Z)}}{\dfrac{N_H N_p(Z)}{D_p(Z)}} \tag{49}$$

Since there is a one to one correspondence between the poles on the s and Z-planes, they can be separated as shown in the numerator of equation (49). The zero order hold used does not in itself contribute any poles to the denominator of equation (49), therefore, equation (49) can be written as above.

Now cancelling the identical poles in equation (49) the final Z-transform necessary to duplicate the continuous action is then

$$D_c(Z) = \frac{N_{cI} N_p(Z)}{D_{cI}(Z) N_H N_p(Z)} \tag{50}$$

Thus, in order to duplicate a continuously controlled system using a digital controller, the controller transfer function must be calculated in this manner, for it is not equivalent to the corresponding Z-transform of the s-transfer function for the continuous controller.

It should be noticed, that in the above derivation, sampling period is an independent factor and can be set by the designer. For this sampling period the Z-transform equation representing the digital controller can be derived which will give the same transient response to a disturbance as the

original continuous controller. Thus, by this method, digital controllers may be designed to give a specified transient response for a maximum sampling interval, the response being specified as that of the continuous system for the same disturbance.

For small sampling periods the Z-transform of the continuous controller, used to represent the digital controller, gives approximately the same response when it is substituted directly. As the sampling periods become larger, this direct substitution approximation becomes less accurate, and from the derivation shown previously in this section, they cannot be expected to be equivalent. Since large sampling periods are desirable, the above method seems particularly suited for the design of digital controllers to replace the analogue controllers in existing control loops. An example illustrating this method is presented in Appendix (A.9.7).

8. RESULTS

1. It has been shown by example, (Appendix A.9.2), that this program can be used for the root locus stability analysis of a continuous system with a dead-time in either, or both the forward or feedback loops. Bode and Nyquist plots can also be derived, (Appendix A.9.3).
2. The root locus calculation was found to be dependable, and gave accurate results for all systems treated. These systems were linear and of three types:
   a. The conventional algebraic transfer function in s.
   b. Pure time delay added to type "a".
   c. Sampled-data systems.
3. Bode and Nyquist subroutines, (Appendix A.1, A.2) were added to the main body of the program and the operation of these subroutines was checked using an example from (18). The plots corresponded exactly with those given in the reference. The resulting plots are shown in Appendix (A.9.3).
4. No use can be made of this program when frequency response data alone is available. The program can only treat a problem when its components are represented as transfer functions in s or Z.
5. Using the Z-transform capability of the program, the transient response of a number of different types of systems can be obtained.

a. Linear, algebraic systems in s.

The transient response for this type of system can be calculated quite accurately. The method used is outlined in sections (7.3.1) and (7.3.2). Accuracy of the transient solution can be very sensitive to the agreement between root loci. To ensure the validity of the transient solution the root loci agreement should be within about one percent for the gain at each point on the root locus, the poles and zeros should agree exactly, and the trajectories of the roots in the complex plane should fall one on the other. Generally, the smaller the sampling period, the closer is the agreement between the two root loci.

b. Linear, algebraic systems in s with dead-time.

The method which is used for the calculation of the transient response requires that the denominator of the closed loop transfer function be factorable to first and second order terms in s. For dead-time in this system this is not possible, therefore, the transient response for this type of system cannot be obtained by the method outlined in Section (7.3.1).

c. Sampled-data system.

No matching of root loci is required in order to find the transient response of this system. It is found by calculating the closed loop transfer function in Z and inverting this transfer function, Sections (7.3.3, 7.3.4).

d. Systems with a digital controller

The transient response of a system such as this can be obtained. Care must be taken when combining the Z-transform polynomial of the controller and that of the rest of the loop. If one sampling period is some multiple of the other, the submultiple sampling method(12) will most likely have to be used. The closed loop transfer function of this would then be inverted to give the transient response.

6. Sampling effect on a system (7.2)

Studies can be made of this effect either through the use of the root locus or the calculation of the transient response of the system to a disturbance.

If the sampler is in the forward loop, the transfer function

$$\frac{C(Z)}{I(Z)} = \frac{G_1(Z)}{1 + G_1(Z)G_2(Z)} \tag{51}$$

is obtained and the transient response is obtained using the method outlined in Section (7.3.3).

If the sampler is in the feedback loop, the Z-transform is

$$C(Z) = \frac{G_1I(Z)}{1 + G_1(Z)G_2(Z)} \tag{52}$$

where the input is not separable from the transfer

function. For this case, the method outlined in Section (7.3.4) must be used.

7. The effect of a digital controller.

The program may be used to study the effect different digital controllers would have on a control loop. Design of a digital controller for a certain loop can be done using information obtained from the root locus plot for the system. An outline of the method is presented in Section (7.4.1) an example is included in the Appendix, (A.9.6).

8. Other uses of the program.

a. Demonstrative for students

The program is easy to use and a student, after approximately a half hour explanation of the data input method, should be able to use the program for Root Locus, Nyquist and Bode calculations. At present, the use of the program in calculating transient response of continuous or sampled-data systems requires more than cursory knowledge of the program. The primary requirement for this, and for the design of digital controllers is knowledge of the Z-transform which will allow the user to make subjective decisions regarding the calculation procedure. These procedures are outlined in Sections (7.3) and (7.4).

b. As a research tool

It could be used for studying the effect that vari-

ations in integral time, loop gain, and in the sampled-data case, sampling periods have on system stability.

The effect of different control configurations could also be studied.

c. "On-Line" Simulation

Most chemical systems, being fairly slow in response, could be simulated by this program as long as they met the following specifications

(1) All equations in the blocks between the output and input are Laplace transformable.

(2) There is no dead-time present in the loop. See Section (7.2).

9. Grid Size Specification

The increments for the grid search used in the root locus calculation determine the information which can be obtained. If the grid is coarse, fewer scan lines cross a locus and less points on it are calculated. With a coarse grid it is also possible to miss a pole or zero. The user should always check this possibility. A missed pole or zero may be spotted easily by noting the magnitude and sign trend in the value of the loop gain given for each point on the locus.

Grid size specification depends largely on the purpose of the calculation. A fine grid may give more points on the root locus but it also means more computer time. If

it is necessary to have all the poles and zeros of the system picked out, a good criteria to follow is to set the grid boundaries large enough to include the largest significant pole or zero. The real axis grid increment should then be specified as approximately one-third the distance between the two closest roots so as to place at least two grid points between them. The lower boundary of the scan area is set to be the real axis as the root locus is symmetrical about the real axis.

10. The Accuracy of the Transient Response Determination

For a continuous system, the accuracy is strongly dependent on the agreement obtained between the two root loci, Section (7.3). Example problems are included in the Appendix, (A.9.4, A.9.5), to illustrate this dependence.

For a digitally controlled system, the transient response obtained may be assumed correct. The calculation requires no matching of root loci,and hence, involves only the inversion of the closed loop transfer function in Z. Caution must be exercised to prevent the use of a sampling period greater than half the bandwidth of the signal. For example, if a sine input were used, the sampling frequency should be at least twice that of the input signal.

11. Accuracy of the Root Locus Calculation

A half interval search is used between scan points for a minimum of 4 iterations. The next interval is then required to be less than 0.005 before the search is discontinued.

12. Program Requirements

a. The transfer function for each block in the block diagram be either in s or Z notation.

b. If a Z-transform calculation is to be made, the form of the s terms in the denominator must be either

$$s + a$$

or

$$s^2 + bs + c$$

c. The block diagram should have no inner loops.

13. Pole-Zero Cancellation

This presents no special computational difficulty. For this case the word pole or zero or nothing at all might be printed out. It is up to the user to check the poles and zeros found by the program and determine the cause if the number printed out does not agree with that expected for the specific problem.

9. CONCLUSIONS

1. This program can be used effectively for the:
   a. Stability analysis of a continuous system.
   b. Stability analysis of a sampled-data system.
   c. Stability analysis of either system with pure time delay.
   d. Transient response determination for a continuous system.
   e. Transient response determination for a sampled or digitally controlled system.
   f. Study of the effect of sampling period in a system containing a sample-hold device.
   g. Demonstration and comparison of Root Locus, Nyquist and Bode Plots as stability analysis tools.
   h. "On Line" simulation of a process provided the specification mentioned in the results, Section (9.8-c), are met.
2. The accuracy of the simulation of a continuous system depends on the match of the root loci for the sampled and continuous cases.
3. The number of points on the root locus, but not the accuracy of these points, is determined by the grid increment set by the user. The accuracy to which these points are determined is independent. For a sampled system the accuracy of the points on the root locus may

be affected by the number of terms used in the series to calculate the Z-transform.

4. The program can be used for the design and specification of a digital controller for a system.
5. The program can be used to design a digital controller to replace a continuous controller which will give the same response as the continuous case.

10. RECOMMENDATIONS

1. Examine the possibility of adding the Share Library Program(23) which determines polynomial roots, to the Control System Analysis program.
2. Further expansion of the program so that the closed loop transfer function can be calculated by the machine for non-unity, multiple feedback systems.
3. Subroutine PDIV be expanded to include disturbances other than the step form currently used, such as sine wave, point input, and square wave.
4. Subroutine ADD be expanded to handle systems other than the unity feedback systems presently handled.
5. Examine the possibility of approximating pure time delays using polynomial approximations for the calculation of the transient response of a system containing a time delay.

11. NOMENCLATURE

K	closed loop gain of the system
q	system number for $G(s)/s=a+ib$ where a and b are numeric
s	independent variable of the Laplace transform
T	sampling interval/period
Z	independent variable of the Z-transform
$D(Z)$	denominator of a Z-transform
$D_c(Z)$	Digital controller transfer function in Z
$e(0^+)$	the value of the sampled input signal at $t = 0^+$
$G(s)$	transfer function in s
$G(Z)$	transfer function in Z
$G_{cL}(s)$	closed loop transfer function
G_{oL}	open loop transfer function
$I^*(s)$	sampled input signal
$\theta(Z)$	Z-transform of a sampled signal
$\theta_i(s)$	general input variable
$\theta_L(s)$	load variable
$\theta_o(s)$	output variable
$\theta^*_i(s)$	sampled general input signal
$\theta^*_o(s)$	sampled output signal

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A P P E N D I X

SUBROUTINE BODE

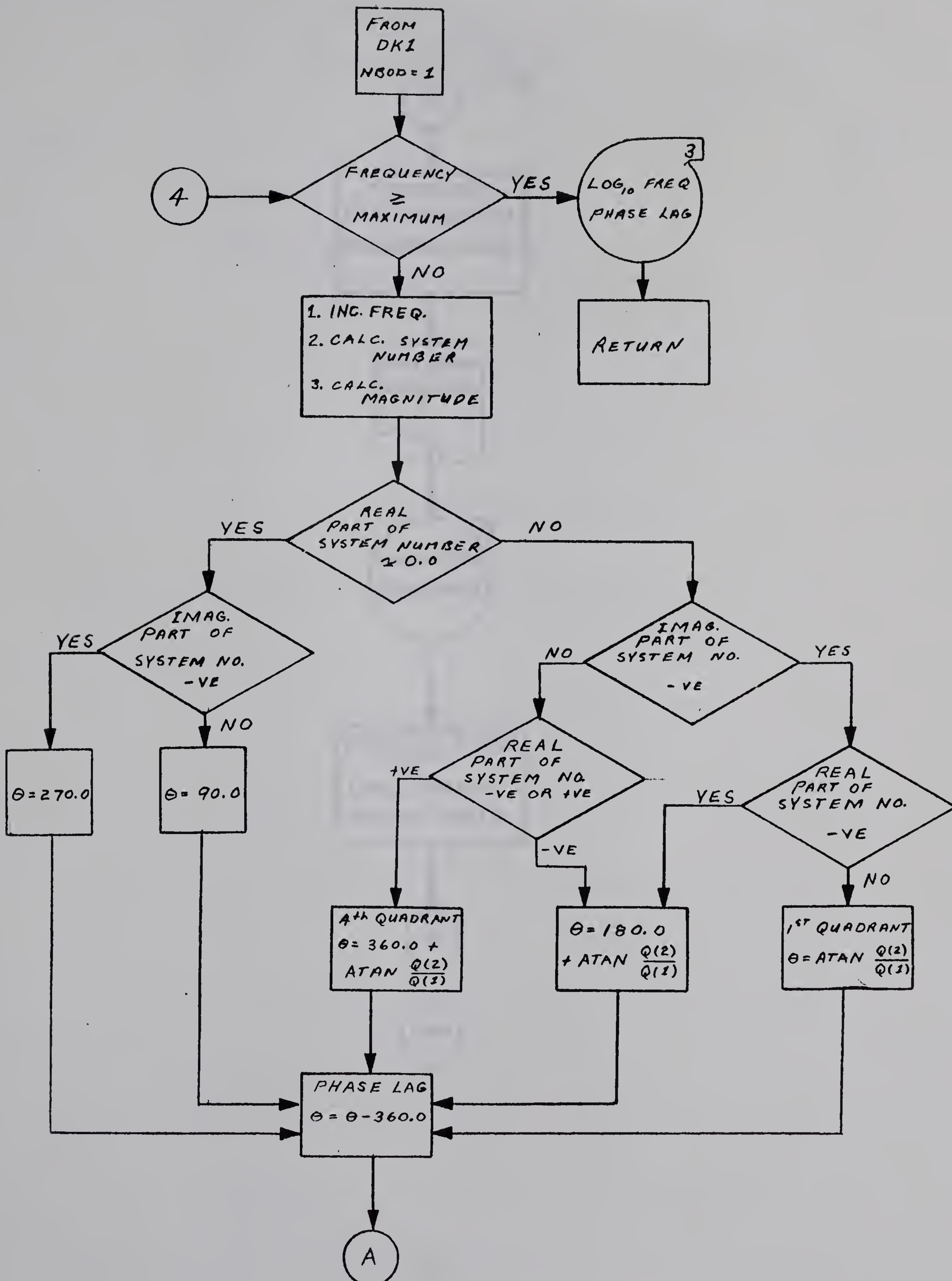
```
      SUBROUTINE BODE (NBOD,OMEGA,DOMEG,OMEGF)
C**************************************************************
C**
C**   THIS SUBROUTINE CALCULATES THE MAGNITUDE AND PHASE
C**   ANGLE GIVEN A TRANSFER FUNCTION WHEN I*OMEGA IS SUB-
C**   STITUTED FOR S.
C**
C**   THE LOG MAGNITUDE IS PRINTED OUT UNDER THE HEADING
C**   MAGNITUDE.  IT IS ALSO PLOTTED ON THE Y-AXIS VS. LOG
C**   FREQUENCY.
C**
C**   THE ANGLE THETA IS PRINTED OUT IN DEGREES.  IT IS ALSO
C**   PLOTTED IN DEGREES.  THETA IS DEFINED AS THE PHASE
C**   LAG OF THE SYSTEM.
C**
C**   DATA     READ IN BY DK1.  FORMAT ...3E10.5,I5..1ST CARD
C**            AFTER M-VECTOR, STARTING IN COLUMN 6, ENDING
C**            WITH C.C. 40.
C**
C**   IF THIS SUBROUTINE IS TO BE USED, THERE MUST BE
C**   SPECIFIED
C**           OMEGA = STARTING FREQUENCY
C**           DOMEG = INCREMENTAL FREQUENCY
C**           OMEGF = FINAL FREQUENCY
C**           NBOD = 1, FLAG FOR THIS PROGRAM. 1 SIGNIFIES ON
C**
C**************************************************************
      DIMENSION E(2,100)
      DIMENSION JO(60),A(500),M(500),S(2,35),Z(2,20,6),Q(2),
     1C(2),D(2),B(
     135,36),CA(36),SUM(35),W(2,10)
      COMMON   JO      , A       , M       , S       , Z       ,
     1W
      COMMON   Q       , C       , D       , X       , B
      REAL OMEGA,DOMEG,OMEGF
      INTEGER NBOD
      SAVE=OMEGA
      X1=-0.1E-10
      X2=1.0E-10
      J=0
      WRITE(6,1)
C**
C**          WRITE TITLE
C**
      NYQ=NBOD
    4 IF (OMEGA .GE. OMEGF) GO TO 5
      N=2
      NH=1
      OMEGA=OMEGA+DOMEG
      S(N,1)=OMEGA
      S(NH,1)=0.0
      CALL SCAN (NYQ)
C**
C**          CALCULATE THE MAGNITUDE
C**
```

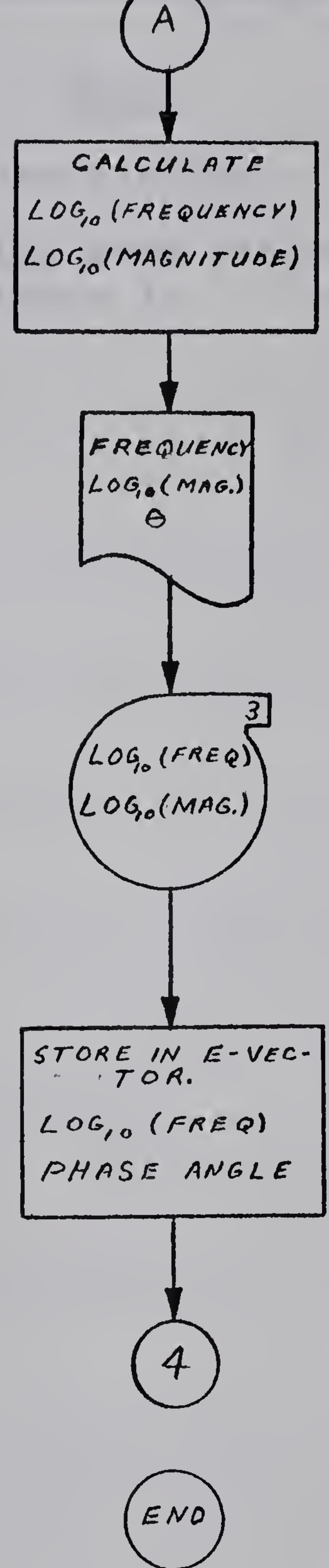
```
      AMAG=SQRT(Q(1)*Q(1)+Q(2)*Q(2))
C**
C**             CALCULATE THE PHASE ANGLE
C**             WHAT QUADRANT ARE WE DEALING WITH
C**
      IF(Q(1) .LT. X1) GO TO 9
      IF(Q(1) .GT. X2) GO TO 9
C**
C**             IS THE ANGLE = 90 OR 270 DEGREES
C**
      IF(Q(2) .LT. 0.0) GO TO 8
      THETA=90.0
      GO TO 14
   8  THETA=270.0
      GO TO 14
   9  IF(Q(2) .LT. 0.0) GO TO 12
      IF(Q(1) .LT. 0.0) GO TO 11
C**
C**             1ST QUADRANT
C**
      THETA=ATAN(Q(2)/Q(1))*57.29578
      GO TO 14
C**
C**             2ND QUADRANT
C**
  11  THETA=180.0+ATAN(Q(2)/Q(1))*57.29578
      GO TO 14
  12  IF(Q(1) .LT. 0.0) GO TO 13
C**
C**             4TH QUADRANT
C**
      THETA=360.0+ATAN(Q(2)/Q(1))*57.29578
      GO TO 14
C**
C**             3RD QUADRANT
C**
  13  THETA=180.0+ATAN(Q(2)/Q(1))*57.29578
C**
C**             PHASE LAG
C**
  14  THETA=THETA-360.0
      BMEGA=ALOG10(OMEGA)
      BMAG=ALOG10(AMAG)
C**
C**             WRITE OUTPUT
C**
      WRITE(6,2) OMEGA,BMAG,THETA
C**
C**   THE RESULTS ARE WRITTEN ON TAPE 3 FOR THE AUTOPLOTTER.
C**   THE MAG. PLOT IS FIRST.  X-AXIS----LOG FREQUENCY
C**                       Y-AXIS----LOG MAGNITUDE RATIO
C**   PHASE ANGLE PLOT   X-AXIS----LOG FREQUENCY
C**                          Y-AXIS---PHASE ANGLE
C**             PLOT LOG AMPLITUDE VS. LOG FREQUENCY.
C**             PLOT PHASE ANGLE VS. LOG FREQUENCY.
```

```
C**
      WRITE (3,3) BMEGA,BMAG
      J=J+1
      E(1,J)=BMEGA
      E(2,J)=THETA
      GC TC 4
    5 CMEGA=SAVE
      WRITE(3,7)
      N=J
      CC 6 J=1,N
    6 WRITE(3,3) E(1,J),E(2,J)
      RETURN
    1 FORMAT (1HL,58X,11HBODE POINTS,//40X,9HFREQUENCY,12X,9
     1HMAGNITUDE,6
     1X,11HPHASE ANGLE)
    2 FORMAT (32X,3E20.8)
    3 FCRMAT (2E13.4)
    7 FORMAT (6H END  )
      END
```

Figure 10

Flow Diagram for Subroutine Bode




A
CALCULATE
LOG10 (FREQUENCY)
LOG10 (MAGNITUDE)
FREQUENCY
LOG10 (MAG.)
θ
3
LOG10 (FREQ)
LOG10 (MAG.)
STORE IN E-VECTOR.
LOG10 (FREQ)
PHASE ANGLE
4
END

APPENDIX A.2

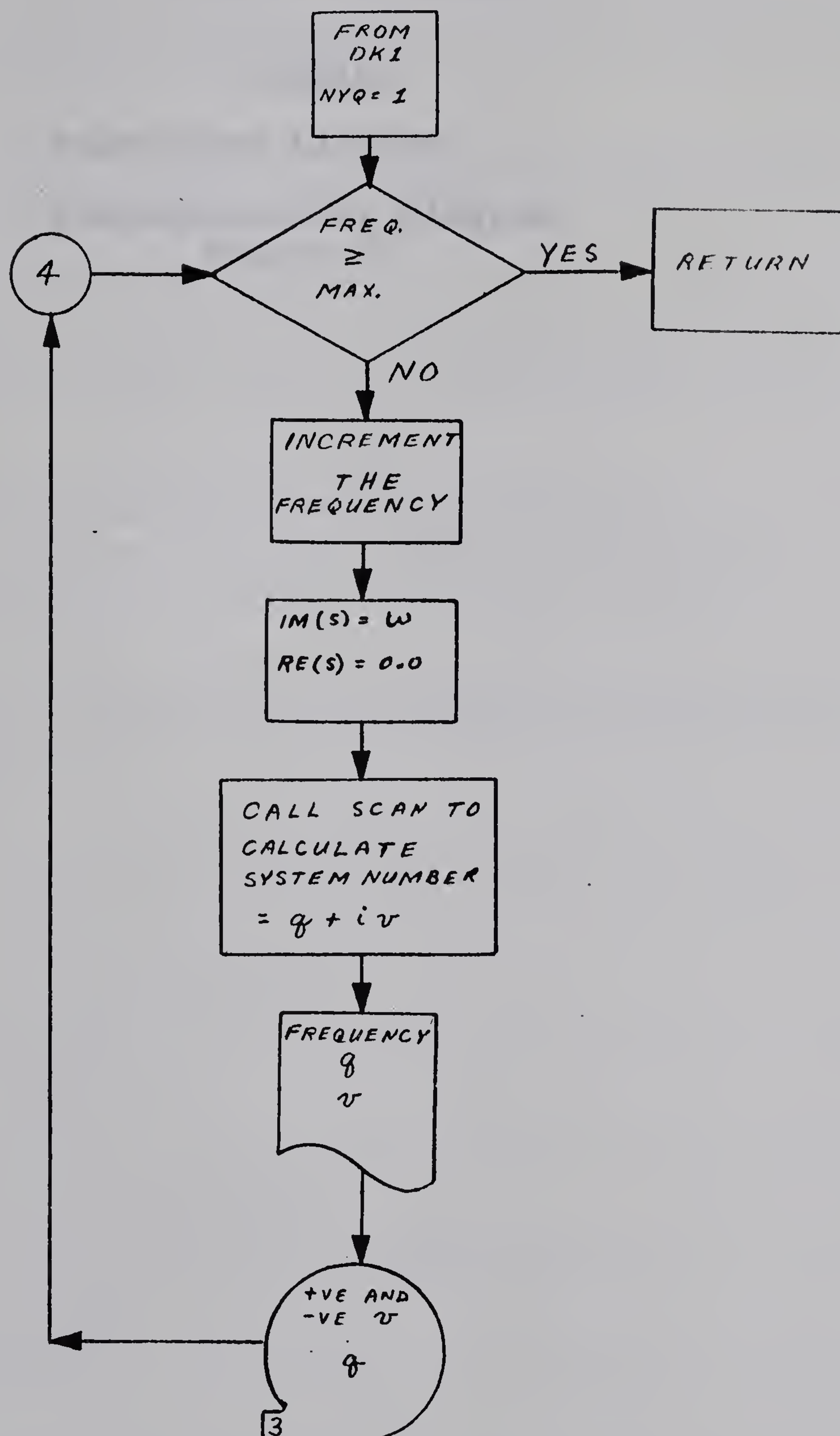
SUBROUTINE ANYQ

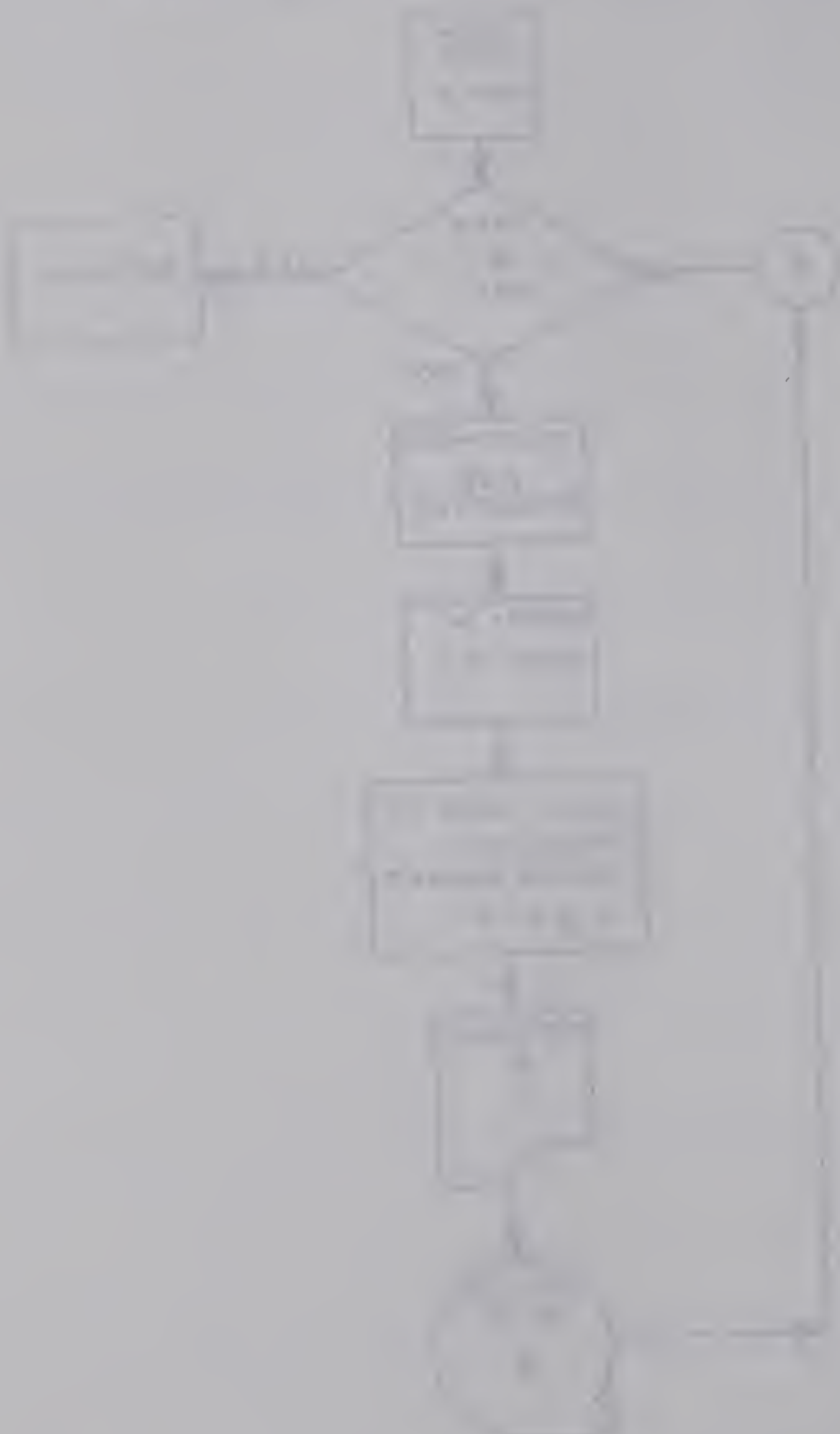
```
      SUBRCUTINE ANYC (NYQ,OMEGA,DOMEG,OMEGF)
C*****************************************************************
C**
C**   THE ACTUAL FRECUENCY IS GIVEN IN THE PRINTOUT UNDER
C**   FREQUENCY.
C**   THE LOG TO THE BASE 10 OF THE FREQUENCY IS PLOTTED ON
C**   THE X-AXIS.
C**
C**   THIS PROGRAM CAN BE USED FOR A NYQUIST PLOT ANALYSIS.
C**
C**   VARIABLES TO BE SPECIFIED
C**           OMEGA = STARTING FREQUENCY
C**           DOMEG = INCREMENTAL FREQUENCY
C**           OMEGF = FINAL FREQUENCY
C**   C.C. 6--C.C. 35 INCL.
C**   FORMAT  3E10.5 FOR THE FIRST THREE VARIABLES.
C**           NYQ   = 1, FLAG FOR THIS SUBROUTINE
C**                  FOR NYQ, INSERT A 1 IN C.C. 5
C**   THE DATA CARD CONTAINING THE ABOVE INFORMATION MUST BE
C**   THE 1ST CARD AFTER THE M-VECTOR CARD OR CARDS.
C**
C*****************************************************************
      DIMENSION JC(60),A(500),M(500),S(2,35),Z(2,20,6),Q(2),
     1C(2),D(2),B(
     135,36),CA(36),SUM(35),W(2,10)
      COMMON   JC      , A       , M       , S       , Z       ,
     1W
      COMMON   Q       , C       , D       , X       , B
      REAL OMEGA,DOMEG,OMEGF
      INTEGER NYQ
      WRITE (6,1)
    4 IF (OMEGA .GE. OMEGF) GO TO 5
      N=2
      NH=1
C**
C**   INCREMENT THE FREQUENCY
C**
      OMEGA=OMEGA+DOMEG
C**
C**   INITIALIZE THE FREQUENCIES.
C**
      S(N,1)=OMEGA
      S(NH,1)=0.0
C**
C**   CALCULATE COMPLEX NUMBER OBTAINED FOR I*OMEGA SUBSTITU
     1TED FOR S.
C**
      CALL SCAN (NYQ)
      G=-Q(2)
C**
C**   WRITE OUTPUT
C**
      WRITE(6,2) OMEGA,Q(1),Q(2)
C**
C**   WRITE ON TAPE 3 FOR AUTOPLOTTER. +VE AND -VE IMAGINARY
     1 PARTS OF TH
```

```
C**    COMPLEX NUMBER AND THE REAL PART. AXIS---Y=IM., X=RE..
C**
       WRITE(3,3) Q(1),Q(2),G
       GO TO 4
   5   NYQ=NYQ+1
       RETURN
  1    FORMAT (1H1,57X,14HNYQUIST POINTS//42X,5HOMEGA,14X,9HR
      1EAL PART,7X,
      19HIMAG PART)
  2    FORMAT (32X,3E20.8)
  3    FORMAT (3E13.4)
       END
```

Figure 11

Flow Diagram for Subroutine ANYQ





APPENDIX A.3

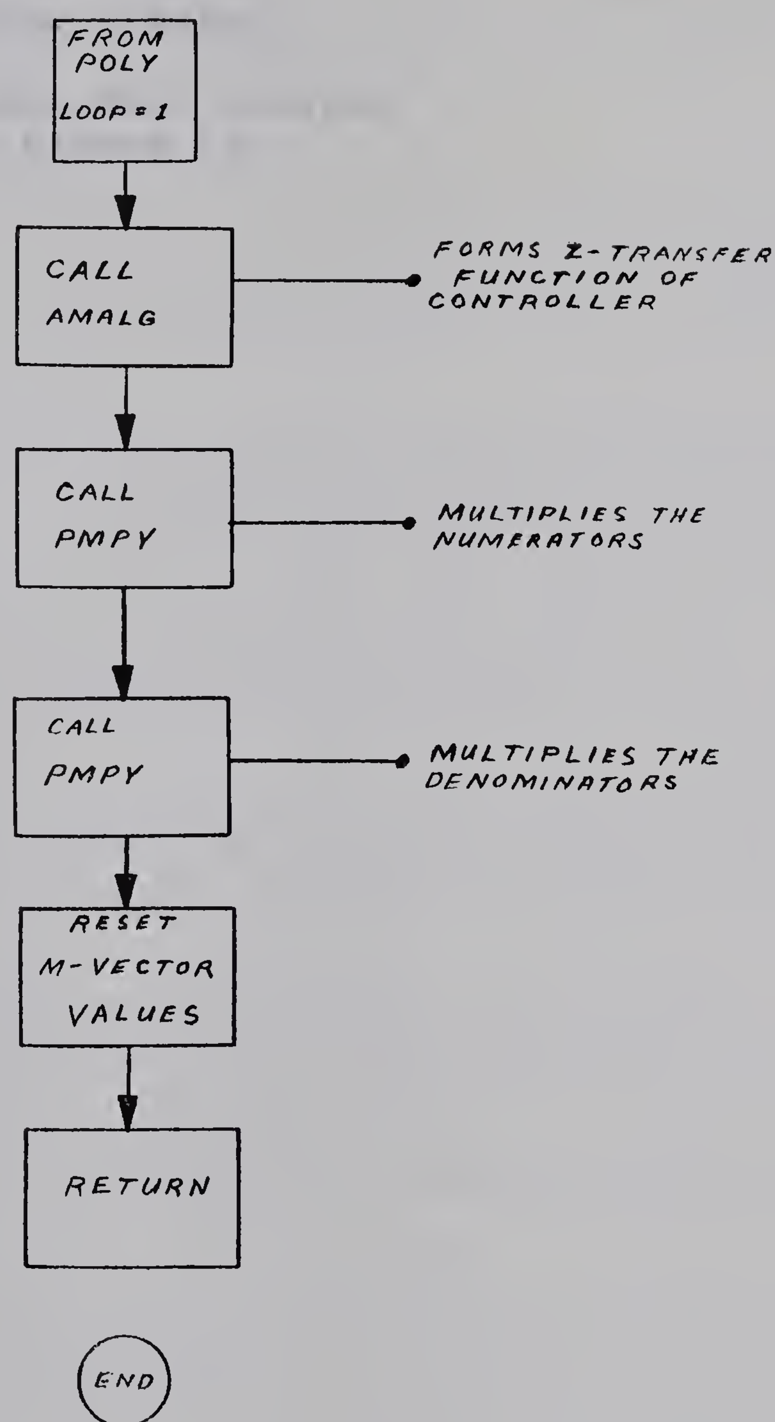
SUBROUTINE MULT

```
      SUBROUTINE MULT (A,ADEN1,NPRO,M)
C*****************************************************************
C**
C**   THIS SUBROUTINE GUIDES THE CALCULATIONS NECESSARY TO
C**   FORM THE FORWARD LOOP TRANSFER FUNCTION, GIVEN A CON-
C**   TROLLER IN Z AND THE PROCESS TRANSFER FUNCTION IN S
C**   AND IS CALLED WHEN LOOP=1.  SEE PDIV.
C**
C**   SUBROUTINE AMALG FORMS THE NUMERATOR AND DENOMINATOR
C**   TERMS OF THE DIGITAL CONTROLLER TRANSFER FUNCTION INTO
C**   ONE POLYNOMIAL EACH.
C**
C**   ADEN=STORAGE VECTOR FOR DENOMINATOR COEFFICIENTS
C**   ANUM=STORAGE VECTOR FOR NUMERATOR COEFFICIENTS
C**   M1=STORAGE VECTOR FOR THE NUMBER OF TERMS IN EACH
C**
C**   SUBROUTINE PMPY IS AN I.B.M. LIBRARY SUBROUTINE
C**   IT SERVES TO MULTIPLY THE TRANSFER FUNCTIONS OF
C**   PROCESS AND CONTROLLER.  THE NUMERATORS FIRST , DEN-
C**   OMINATORS SECOND.  SEE PMPY FOR VARIABLE LIST
C**
C*****                VARIABLE LIST
C**   A---STORAGE AREA FOR PROCESS NUMERATOR
C**   ADEN1---STORAGE AREA FOR PROCESS DENOMINATOR
C**   NPRO---NUMBER OF TERMS IN THE DENOMINATOR
C**   M---M-VECTOR
C**
C*****************************************************************
      DIMENSION A(500),ADEN1(36),ADEN(20),ANUM(20),M1(10),M(
     1500),Z(36),A
     1NUM1(20)
C**
C**   CALCULATE THE CONTROLLER NUMERATOR AND DENOMINATOR
C**   POLYNOMIALS
C**
      CALL AMALG(ADEN,ANUM,M1)
      NPRO=IABS(NPRO)
      DO 1 I=1,NPRO
      ANUM1(I)=A(I+400)
    1 A(I+400)=0.0
C**
C**   MULTIPLY THE NUMERATORS
C**
      CALL PMPY(Z,IDIMZ,M1(2),ANUM,NPRO,ANUM1)
      DO 2 I=1,IDIMZ
    2 A(I+400)=Z(I)
      M(2)=IDIMZ-1
      M(1)=1
C**
C**   MULTIPLY THE DENOMINATORS
C**
      CALL PMPY(Z,IDIMZ,M1(4),ADEN,NPRO,ADEN1)
      DO 3 I=1,IDIMZ
    3 ADEN1(I)=Z(I)
C**
```

```
C**    RESET M-VECTOR VALUES
C**
       M(3)=1
       M(4)=IDIMZ-1
       NPRO=IDIMZ
       RETURN
       END
```

Figure 12

Flow Diagram for Subroutine MULT



APPENDIX A.4

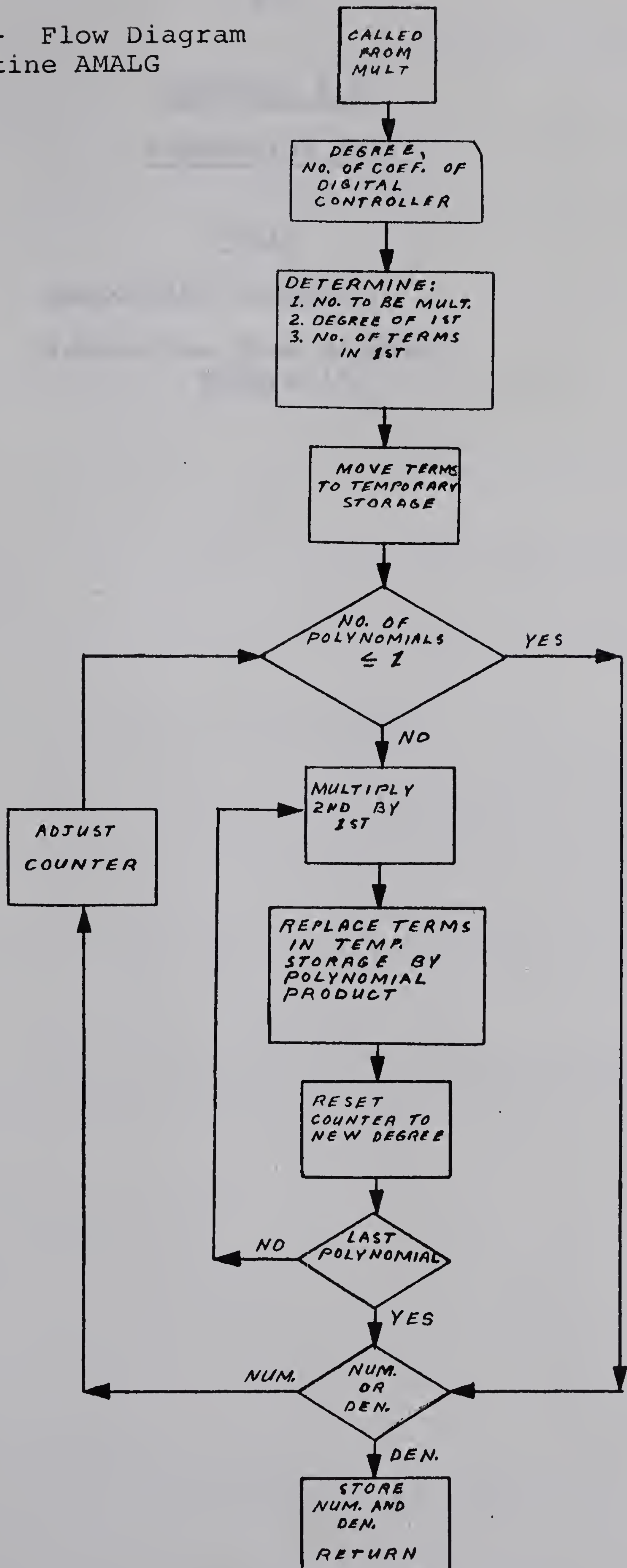
SUBROUTINE AMALG

```
      SUBRCUTINE AMALG(ADEN,ANUM,M1)
      CIMENSION ADEN(20),ANUM(20),M1(10),A1(40),DG1(40),B1(3
     1,4)
C*************************************************************
C**
C**   THIS ROUTINE IS CALLED FROM SUBROUTINE MULT. ITS PUR-
C**   POSE IS TO MULTIPLY THE TERMS IN THE NUMERATOR AND
C**   DENCMINATOR OF THE DIGITAL CONTROLLER TRANSFER
C**   FUNCTION.
C**   CNE TERM IN THE NUMERATOR AND ONE IN THE DENOMINATOR
C**   IS FCRMED FROM THESE.
C**   IN ESSENCE, GIVEN THE NUMBER OF, AND THE DEGREE OF
C**   EACH OF THE, PCLYNOMIALS, THIS ROUTINE WILL SUCCES-
C**   SIVELY MULTIPLY THEM TOGETHER TO ARRIVE AT THE
C**   POLYNOMIAL PRODUCT.
C*****               INPUT DESCRIPTION
C**   THE INFORMATION CONCERNING THE DIGITAL CONTROLLER TO
C**   BE INSERTED IN THE LOOP IS READ IN FROM THIS SUBROU-
C**   TINE.  IF LOOP IS NOT SET = 1 (SEE SUBROUTINE PDIV),
C**   THIS SUBROUTINE IS NOT REACHED AND NC DATA IS READ.
C*****               DATA INPUT
C**   FOR LCOP SEE PDIV
C**   THE NUMBER OF POLYNOMIALS AND THE DEGREE OF EACH MUST
C**   BE READ IN FOR BOTH NUMERATOR AND DENOMINATOR USING
C**   FORMAT 10I3.  THE NUMERATOR DESCRIPTION PRECEDES THE
C**   DENOMINATOR DESCRIPTION. IF THERE WERE THREE POLYNOM-
C**   IALS IN THE NUMERATOR OF 1ST,2ND, ANC, 1ST ORDER RES-
C**   PECTIVELY THE INPUT INFORMATION WOULC APPEAR AS
C**   3  1  2  1.  THE DENOMINATOR DESCRIPTION IS THE SAME
C**   AND COMES IMMEDIATLEY AFTER THE NUMERATOR.  ORDERED
C**   FROM RIGHT TO LEFT.
C**      THE COEFFICIENTS ARE ENCODED IN THE SAME MANNER AS
C$$   THOSE OF THE A-VECTOR FOR AN ORDINARY S-PLANE ROOT
C**   LOCUS DATA INPUT. THEY ARE ENTERED IN ORDER FROM RIGHT
C**   TO LEFT.
C*****               VARIABLE  DEFINITIONS
C**   ANUM(I)=STORAGE AREA FOR POLYNOMIAL DENOMINATOR
C**   ANUM(I)=STORAGE AREA FOR POLYNOMIAL NUMERATOR
C**   M1(J)=NUM. AND DEN. DESCRIPTION VECTOR
C**                   PROGRAN LIMITATIONS
C**   PRODUCT NUMERATOR OR DENOMINATOR DEGREE MUST BE LESS
C**   THAN 19
C**   TEN M1 VECTOR ENTRIES IS THE MAXIMUM
C**
C*************************************************************
      READ(5, 100) (M1(I),I=1,10)
      NA=0
      MH=M1(1)
      DO 1 I=1,MH
    1 NA=NA+M1(I+1)
      NTN=NA+M1(1)
      NA=0
      NB=M1(1)+2
      NB1=NB+M1(NB)-1
      DO 2 I=NB,NB1
```

```
  2     NA=NA+M1(I+1)
        NT=NA+M1(NB)+NTN
        READ(5,101)(A1(I),I=1,NT)
        NCOUT=0
        JR=0
C**        COUNTER FOR COEFFICIENT DATA
        JM=0
 20     JM=JM+1
        NCOUT=NCOUT+1
        NP=M1(JM)-1
C**        NUMBER OF POLYNOMIALS TO BE MULTIPLIED
 12     JM=JM+1
        NT1=IABS(M1(JM))
C**        DEGREE OF FIRST POLYNOMIAL
        NT=NT1+1
C**        NUMBER OF TERMS IN FIRST POLYNOMIAL
        DO 15 I=1,NT
        JR=JR+1
 15     DG1(I)=A1(JR)
C**        MOVE TERMS OF FIRST POLYNOMIAL FROM A1 VECTOR TO
C**        TEMPORARY STORAGE
        IF(NP .LT. 1) GO TO 24
        DO 19 IB=1,NP
C**        SET UP TEST TO DETERMINE IF MORE MULTIPLICATION
C**        IS NECESSARY
        JM=JM+1
        NT2=IABS(M1(JM))+1
C**        NUMBER OF TERMS IN SECOND POLYNOMIAL
        DO 16 I=1,NT2
        JR=JR+1
        DO 16 J=1,NT
 16     B1(I,J)=DG1(J)*A1(JR)
C**        MULTIPLY FIRST POLYNOMIAL COEFFICIENTS BY NEXT
C**        POLYNOMIAL COEFFICIENTS
        L=NT1+NT2
C**        NUMBER OF TERMS IN COMBINED POLYNOMIAL
        DO 17 K=1,L
 17     DG1(K)=0.0
        DO 18 I=1,NT2
        DO 18 J=1,NT
        K=I+J-1
C**        COEFFICIENT LOCATION
 18     DG1(K)=DG1(K)+B1(I,J)
C**        REPLACE POLYNOMIAL TERMS IN TEMPORARY STORAGE BY
C**        LAST COMBINED POLYNOMIAL PRODUCT
        NT1=NT1+NT2-1
C**        RESET COUNTER TO DEGREE OF COMBINED POLYNOMIAL
 19     NT=NT1+1
 24     IF(NCOUT .GE. 2) GO TO 22
        M1(1)=1
        DO 21 J=1,NT
        ANUM(J)=DG1(J)
 21     DG1(J)=0.0
        M1(2)=NT
        GO TO 20
```

```
22    DO 23 J=1,NT
      ADEN(J)=DG1(J)
23    DG1(J)=0.0
      M1(3)=1
      M1(4)=NT
100   FORMAT(10I3)
101   FORMAT(9E8.5)
      RETURN
      END
```

Figure 13 - Flow Diagram for Subroutine AMALG



APPENDIX A.5

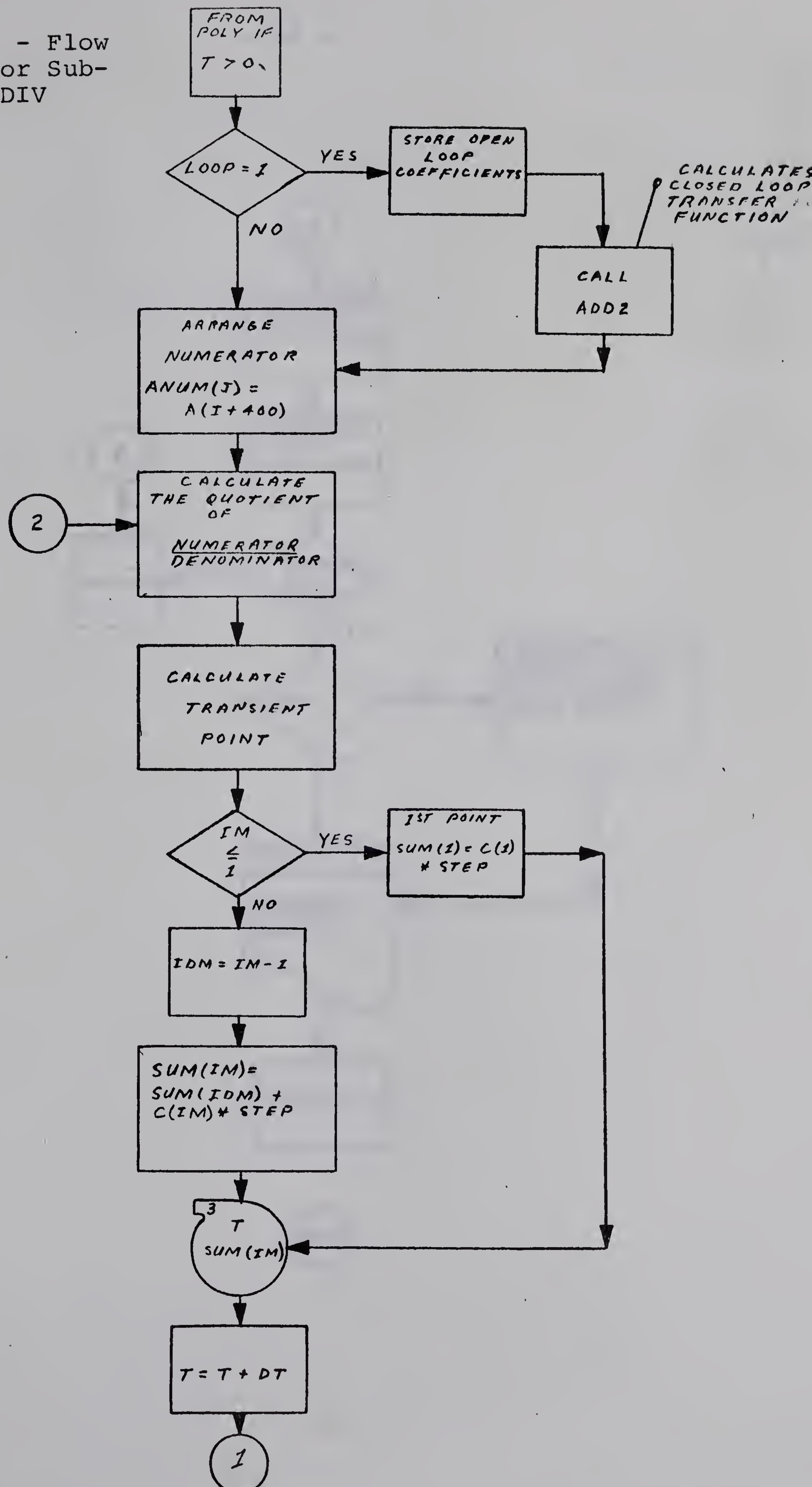
SUBROUTINE PDIV

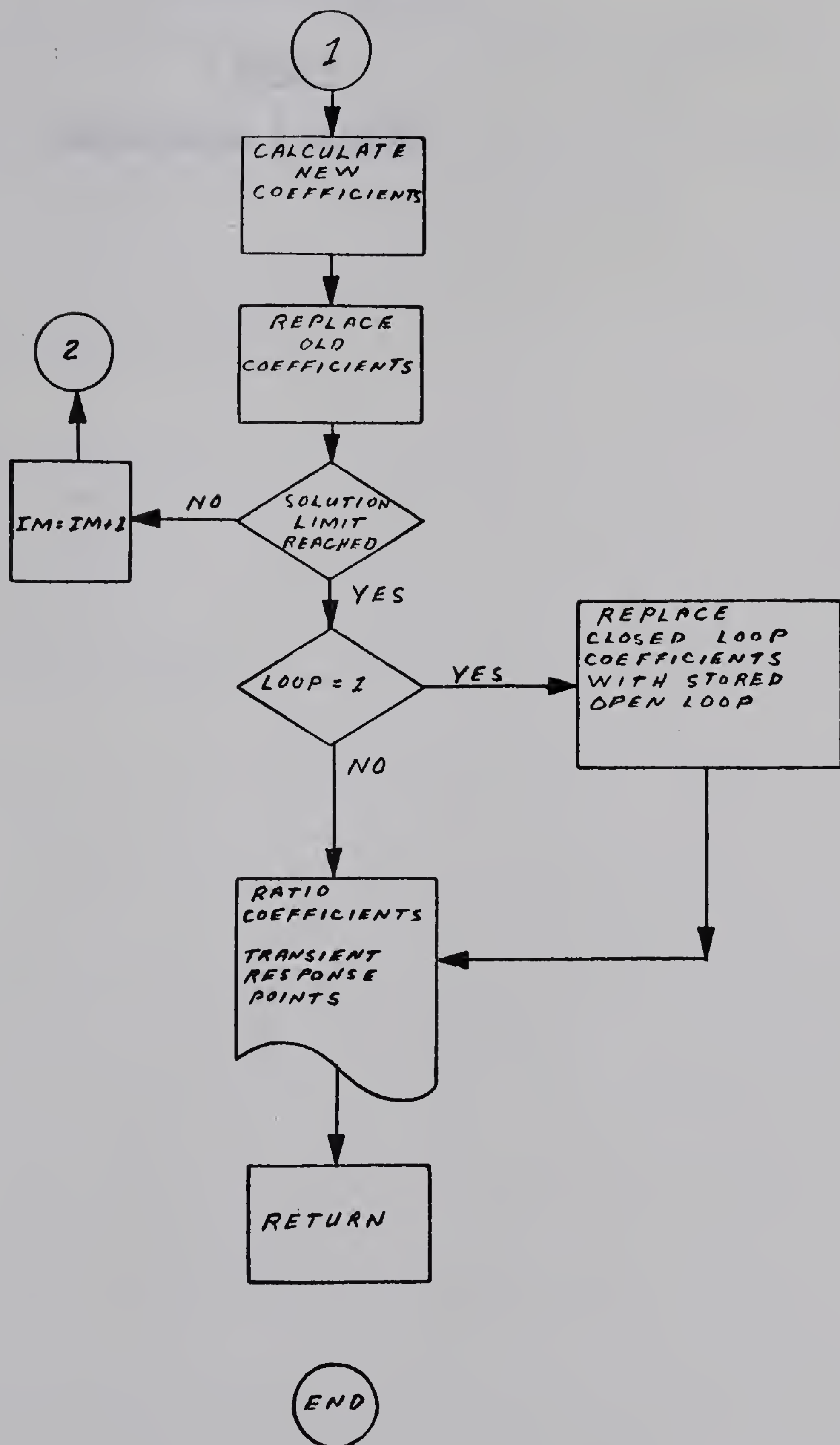
```
      SUBRCUTINE PDIV(A,ADEN,N,DT,T,FMT,STEP,LOOP)
C*****************************************************************
C**
C**   THIS SUBROUTINE IS USED FOR THE INVERSION OF A Z-TRAN-
C**   SFORM. IT CIVIDES THE NUMERATOR OF THE Z-TRANSFORM BY
C**   ITS CENOMINATOR.
C**   THE Z-TRANSFORM TREATED HERE IS THAT OF THE RATIO OF
C**   CUTPUT TO INPUT, THUS THE EVALUATION OF THE TRANSIENT
C**   RESPCNSE CONSISTS OF SUMMING THE PRODUCT OF THE Z-TRAN
C**   -SFORM POLYNOMIAL COEFFICIENTS AND THE TIME VALUE OF
C**   THE INPUT FUNCTION AT THE VARIOUS SAMPLING INSTANTS.
C*****              INPUT VARIABLES
C**         DT = INCREMENT IN TIME EQUAL TO THE SAMPLING
C**                TIME.
C**          T = MAXIMUM TIME FOR WHICH SOLUTION IS REQUIRED
C**     FMT(1) = UNITS OF TIME (MIN.,HRS.,SECS.)
C**       STEP = SIZE OF STEP USED TO PERTURB THE SYSTEM
C*****              PROGRAM   LIMITATIONS
C**   PROGRAM DIMENSIONING ALLOWS FOR NO MORE THAN 450 POINT
C**   S ON THE TRANSIENT CURVE.
C*****              CATA INPUT
C**   CT,T,FMT(1) ARE ENTERED ON THE FIRST CARD AFTER THE M-
C**   VECTCR CARC.  FORMAT IS 2E10.5,A6,  START AT C.C. 46
C**   END AT  C.C. 71.
C**   M-VECTOR CARD.  START WITH STEP IN C.C. 1 WITH FORMAT
C**   E10.5,I5
C**   LCOP AND STEP ARE READ IN DK1.
C*****              CLOSED LOOP ROUTINE
C**   THE FLAG LCOP IS USED TO ENTER THIS ROUTINE.  WHEN
C**   LCOP = 1 AND T IS NOT = 0, THE ALGORITHM FOR CALCULAT-
C**   ING THE CLOSED LOOP TRANSFER FUNCTION OF THE SYSTEM,
C**   SUBRCUTINE ADD, IS ENTERED AND USING THIS TRANSFER
C**   FUNCTION THE TRANSIENT RESPONSE IS CALCULATED.
C**
C**   IF LCOP IS NOT SET = 1, NO MULTIPLICATION OF THE PRO-
C**   CESS TRANSFER FUNCTION BY THE CONTROLLER FUNCTION
C**   CCCURS.
C**   IF T = 0, PCIV IS NOT ENTERED.
C**
C*****************************************************************
      CIMENSION ANUM(500),ADEN(36),C(450),FMT(12),A(500),SUM
     1(45C)
      INTEGER N
      WRITE(6,7)
      WRITE(6,8) CT,FMT(1)
      WRITE(6,16)
      N=IABS(N)
      C=T/CT
      IMT=C
      IN=N+1
      CO 1 I=1,N
      J=IN-I
    1 ANUM(J)=A(I+40C)
C**
C**     DIVISION OF CNE POLYNOMIAL BY ANOTHER
```

```
C**
      IF(LCOP .NE. 1) GC TO 23
      NT=N
C**
C**   STORE OPEN LOOP VECTOR
C**
      CO 24 J=1,NT
 24   ANUM(J+4CC)=ADEN(J)
C**
C**   CALCULATE CLOSED LOOP TRANSFER FUNCTION
C**
      CALL ADD(A,ADEN,N)
 23   T=0.C
      IM=1
C**
C**      CALCULATE THE QUOTIENT
C**
  2   I=IN
      C(IM)=ANUM(1)/ADEN(N)
C**
C**   CALCULATE POINTS ON TRANSIENT CURVE.
C**
      IF(IM .LE. 1) GC TO 20
      IDM=IM-1
      SUM(IM)=SUM(IDM)+C(IM)*STEP
      GC TC 18
 20   SUM(1)=C(1)*STEP
      IDM=IM-1
 18   WRITE(3,10) T,SUM(IM)
C**
C**      INCREMENT THE TIME.
C**
      T=T+CT
C**
C**      CALCULATE THE NUMERATOR.
C**
      CO 3 J=1,N
      I=I-1
  3   ANUM(J)=ANUM(J)-ADEN(I)*C(IM)
C**
C**   REPLACE OLD NUMERATOR COEFFICIENTS WITH THOSE JUST
C**   CALCULATEC.
C**
      ANUM(IN)=C.0
      CO 13 J=2,IN
 13   ANUM(J-1)=ANUM(J)
C**
C**   CHECK FOR SOLUTION LIMIT.
C**
      IF(IM .GE. IMT) GC TO 11
      IM=IM+1
      GC TC 2
 11   IF(LCOP .NE. 1) GC TO 22
      LCOP=LCOP +2
C**
```

```
C**   RESTCRE OPEN LCCP VECTOR IN ORIGINAL POSITION IN
C**   PREPARATICN FOR ROOT LOCUS CALCULATICN.
C**
      CO 21 J=1,NT
      ADEN(J)=ANUM(J+400)
 21   ANUM(J+4C0)=0.C
 22   WRITE(6,6)(C(J),J=1,IM)
      WRITE(6,15) STEP
      WRITE(6,17) (SUM(I),I=1,IM)
      WRITE(3,14)
 14   FORMAT(6H END   )
  12  FORMAT (1H1)
  5   FORMAT(5E15.5)
  6   FORMAT(6E2C.6/)
  7   FORMAT(1H1,48X,34HINSTANTANEOUS TIME DOMAIN RESPONSE/5
     14X,23HFOR SU
     1CCESIVE MULTIPLES/63X,6HOF TAU)
  8   FORMAT(1H3,5X,34HTIMES IN MULTIPLES CF TAU.  TAU = ,F6
     1.3,A6)
  9   FORMAT(2E1C.5,A6)
 10   FORMAT(2E13.5)
 15   FORMAT(1HJ,5X,52HRESPONSE CURVE POINTS FOR A STEP INPU
     1T. STEP SIZE
     1 = ,F6.3)
 16   FORMAT(1HK,5X,35HZ-TRANSFORM POLYNOMIAL COEFFICIENTS/)
 17   FORMAT (1HJ,6E20.6)
      RETURN
      END
```

Figure 14 - Flow Diagram for Subroutine PDIV




1
CALCULATE NEW COEFFICIENTS
REPLACE OLD COEFFICIENTS
2
IM=IM+1
NO
SOLUTION LIMIT REACHED
YES
LOOP = 1
YES
REPLACE CLOSED LOOP COEFFICIENTS WITH STORED OPEN LOOP
NO
RATIO COEFFICIENTS
TRANSIENT RESPONSE POINTS
RETURN
END

APPENDIX A.6

SUBROUTINE PMPY

```
      SUBROUTINE PMPY(Z,IDIMZ,IDIMX,X,IDIMY,Y)
C*****************************************************************
C**
C**   THIS SUBROUTINE MULTIPLIES TWO POLYNOMIALS TOGETHER.
C*****                VARIABLE DEFINITIONS
C**
C**   Z---VECTOR OF RESULTANT COEFFICIENTS, ORDERED FROM
C**       SMALLEST TO LARGEST POWER.
C**   IDIMZ---DIMENSION OF Z (CALCULATED)
C**   X---VECTOR OF COEFFICIENTS FOR FIRST POLYNOMIAL,
C**       ORDERED FROM SMALLEST TO LARGEST POWER
C**   IDIMX---DIMENSION OF X (DEGREE IS IDIMX-1)
C**   Y---VECTOR OF COEFFICIENTS FOR SECOND POLYNOMIAL,
C**       ORDERED FROM SMALLEST TO LARGEST POWER
C**   IDIMY---DIMENSION OF Y (DEGREE IS IDIMY-1)
C**
C*****************************************************************
      DIMENSION Z(36),X(20),Y(20)
      IF(IDIMX*IDIMY)10,10,20
 10   IDIMZ=0
      GO TO 50
 20   IDIMZ=IDIMX+IDIMY-1
      DO 30 I=1,IDIMZ
 30   Z(I)=0.0
      DO 40 I=1,IDIMX
      DO 40 J=1,IDIMY
      K=I+J-1
 40   Z(K)=X(I)*Y(J)+Z(K)
 50   RETURN
      END
```

APPENDIX A.7

SUBROUTINE ADD

```
      SUBRCUTINE ADD(A,ADEN1,N2)
C***********************************************************************
C**
C**   THIS SUBROUTINE IS CALLED FROM PDIV
C**   IT ACDS TWO POLYNOMIALS TO GET THEIR SUM
C**   IT IS USEC AT PRESENT TO CALCULATE THE CLOSED LOOP
C**   TRANSFER FUNCTICN OF A LOOP WITH UNITY FEEDBACK
C*****             VARIABLE DEFINITION
C**   A---STORAGE AREA FOR THE NUMERATOR OF THE FORWARD LOOP
C**          TRANSFER FUNCTION
C**   ADEN1---STORAGE VECTOR FOR THE DENOMINATOR
C**   N2---THE DEGREE OF THE LARGEST POLYNOMIAL. IT IS
C**          ALWAYS THE CENOMINATOR WHEN USED FOR THE ABCVE
C**          PURPOSE
C**
C***********************************************************************
      CIMENSION A(500),ADEN1(36)
      NT=IABS(N2)+1
      DO 1 I=1,NT
    1 ADEN1(I)=ADEN1(I)+A(I+400)
      RETURN
      END
```

APPENDIX A.8

ORIGINAL PROGRAM, SUBROUTINE MODICIATIONS

1. Subroutine DK1
   a. A programming error was corrected so that titles for the various problems could be read in with the problem. A change of format was necessary.
   b. Statements were added so that output for the computing system's autoplotter could be obtained. This made it possible to get graphical output for the solution.
   c. New read statements were added to accommodate subroutines BODE, ANYQ, PDIV, and MULT.
   d. Modified so that subroutine POLY is called, whenever a transient response calculation is called for.
   e. Statements added to obtain the Z-plane root locus.
2. Subroutine POLY
   a. Changed to print out a second order polynomial under both the factored polynomial and the polynomial headings.
   b. The subroutines PDIV and MULT are called from this subroutine.
3. Subroutine OPUT - Modified to:
   a. Write out Z-plane root locus points when called for by the user.

b. Write on Tape 3 for the autoplotter when graphs for the s or Z-plane root loci are requested. These plots are not available for the same run.

4. Subroutine SCAN - Modified to:

a. Contain the branch to subroutine NYQ which calculates points for the Nyquist Plot.

APPENDIX A.9.1

EXAMPLE PROBLEMS AND SOLUTIONS.

TRANSFER FUNCTION INDEX

Appendix Section	Transfer Functions
A.9.2	$G_{oL}(s) = \dfrac{30(s+2.0)}{s(s-3.0)(s+10.0)} e^{-sT}$
A.9.3	$G_{cL}(s) = \dfrac{30(s+2.0)}{s(s-3.0)(s+10.0)}$
A.9.4	$G_{oL}(s) = \dfrac{3.6(0.2s+1)}{0.2s(5.0s+1)}$
	$G_{cL}(s) = \dfrac{0.72(s+5.0)}{s^2 + 0.92s + 3.6}$
	$e^{sT}H_1(s)G_{eL}(s) = \dfrac{e^{sT}}{T}\left(\dfrac{1-e^{-sT}}{s}\right)^2\left(\dfrac{0.72(s+5.0)}{s^2+0.92s+3.6}\right)$
A.9.5	$G_{oL}(s) = \dfrac{3.6(10s+1)}{10s(s+1)(5.0s+1)}$
	$G_{cL}(s) = \dfrac{0.072(10s+1)}{(s+0.0875)(s^2+1.1125s+0.8226)}$
	$e^{sT/2}H_o(s)G_{eL}(s)$
	$= e^{sT/2}\left(\dfrac{1-e^{-sT}}{s}\right)\left(\dfrac{0.072(10s+1)}{(s+0.0875)(s^2+1.1125s+0.8226)}\right)$

A.9.6 $$H_o(s)G_p(s) = \frac{1-e^{-st}}{s} \frac{0.1}{(s+1)(s+0.2)}$$

First Example

$$D_c(Z) \left[Z(H_o(s)G_p(s)\right] = \left(\frac{Z-0.834}{Z-1.0}\right) Z \left[\frac{1-e^{-sT}}{s} \frac{0.1}{(s+1)(s+0.2)}\right]$$

Second Example

$$D_c(Z) \left[Z(H_o(s)G_p(s)\right]$$

$$= \left[\frac{(Z-0.95)(12)}{(Z-1.0)}\right] Z \left[\frac{1-e^{-sT}}{s} \frac{0.1}{(s+1)(s+0.2)}\right]$$

A.9.7 $$D(s)G_p(S) = \frac{s+0.1}{s(s+1)(s+0.2)}$$

$$H_o(s)G_p(s) = \frac{0.1}{(s+1)(s+0.2)}$$

$$D_c(Z)\ Z\left[H_o(s)G_p(s)\right] =$$

$$\left[\frac{0.59557Z^2-0.539112Z-0.00269}{(Z-1)(0.034278Z+0.023014)}\right] Z \left[\frac{1-e^{-sT}}{s} \frac{0.144}{(s+1)(s+0.2)}\right]$$

APPENDIX A.9.2

ROOT LOCUS PLOT FOR A SYSTEM WITH PURE TIME DELAY

Purpose:

To illustrate the procedure used when dealing with a system containing a pure time delay.

To illustrate the effect of pure time delay on a system through the comparison of root loci of the system for various delays.

Transfer Functions:

Without Delay

$$G_{oL}(s) = \frac{30(s+2.0)}{s(s-3.0)(s+10.0)}$$

With Delay

$$G_{oL}(s) = \frac{30(s+2.0)}{s(s-3.0)(s+10.0)} e^{-sT}$$

Outline:

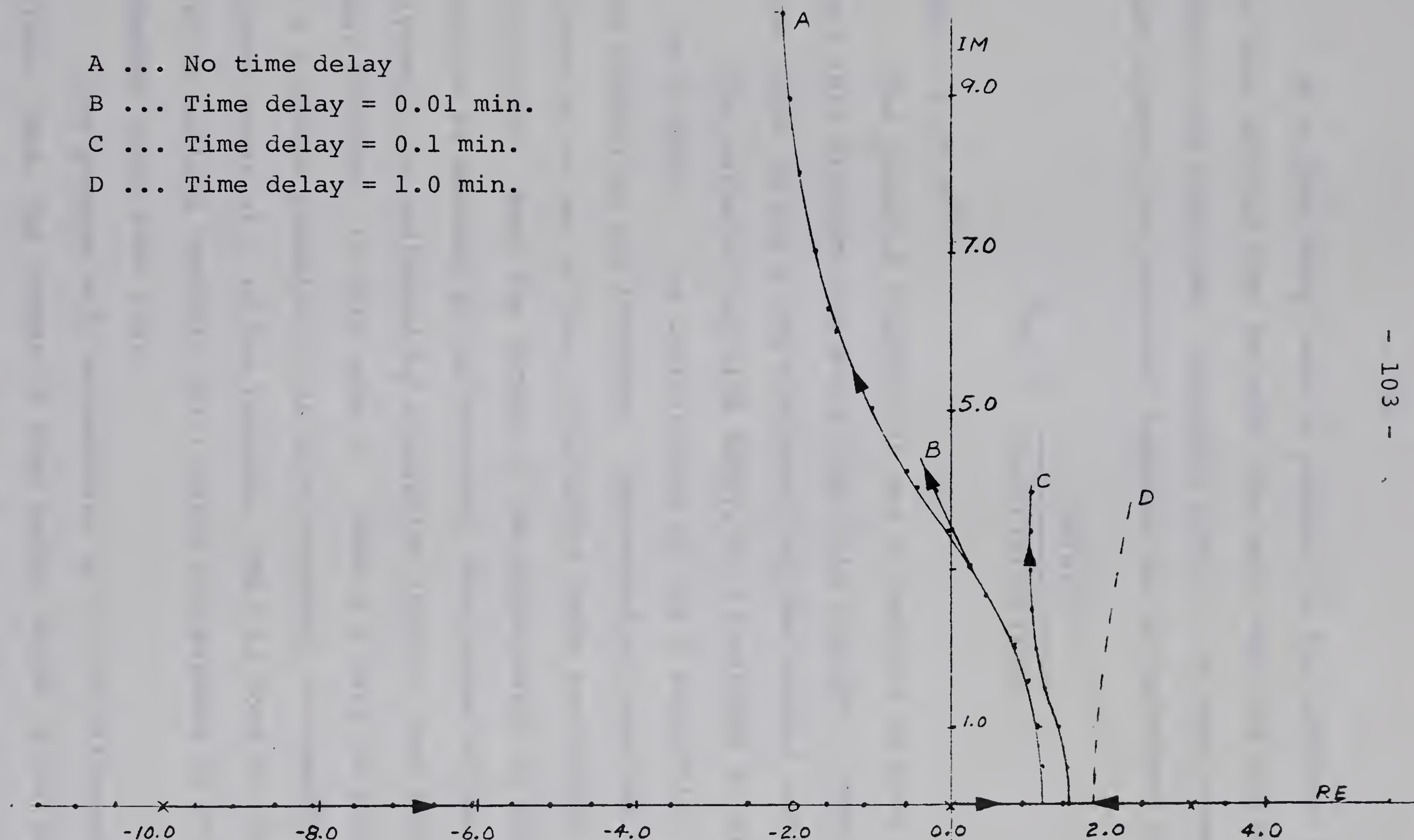
The input data used for this problem is shown in order of input under the heading INPUT DATA on page 105. The input format is shown for each data group in a data set. The control card data and the M-vector data are printed out, in expanded form, by subroutine DPRINT for each run. This provides the user with a check of the input data. Further expansion of subroutine DPRINT

could be made so that the transfer function or functions of the problem is also printed out. This expanded version of the control cards is in itself a shortened description and for further information the user is referred to (24).

The root locus for this system without the time delay is presented by Schilling(18). Schilling's diagram and the one obtained through the use of the C.S.A. program agree exactly. The root locus for the system without the added delay is presented so a comparison may be made between its root locus and those of the same system with different delays added to it.

Time delays of 1, 0.1 and 0.01 minutes were added to the transfer function for the comparison. The results obtained in the form of root locus plots are shown in Graph (1). The trend of the root locus towards that of the system containing no time delay as the delay is decreased is apparent. This type of change with the variation of the time delay is expected and serves to verify the applicability of the program to this type of problem.

It should be noticed that the transfer function contains a positive pole which imparts some peculiarities to the root locus diagram for this system, for there is a lower limit of loop gain, $K = 1.39$, for which the system is stable. Further mention of this will be made in the Appendix Section A.9.3.



Graph 1

Root Loci for Pure Time Delay

If a time delay term is present in the problem, four more data entries must be made than were made for the problem without this dead-time, (Appendix A.9.3). To make these entries clearer, the transfer function can be converted to:

$$G_{oL} = \frac{30(s+2)}{s(s-3)(s+10)(Z+0)}$$

where $Z = e^{sT}$.

The transfer function is now a function of both s and Z and this dictates the extra four data entries. These entries occur in the A and M vectors and the control card.

The value of the time delay, T, is entered in A(7) in the A-vector. The coefficients of the Z-polynomial are also entered in the A-vector. The method of entering these differs in no way to that of entering those pertaining to s polynomials. When the format of the denominator of this function is encoded in the M-vector, the degree of the Z-polynomial is indicated by a negative integer, see INPUT DATA under M-Vector, in this case -1. When a Z-term is included in a transfer function it is also necessary to specify the highest power of Z in the problem. This is done on the control card by setting variable Jo(7) which corresponds to F7 in the expanded input data list.

The program will accommodate up to six different time delays, thus, the number of time delays which it can expect must be set in JO(8) or F8 in the expanded input data list. For this case JO(8) is equal to one.

INPUT DATA

```
CCNTRCL CARC CATA (FCRMAT 24I3)

 2C  8  1  C  C  1  1  1  0  0 -2  0

TITLE

 EXAMPLE SCHILLING PG. 214. DEAC-TIME = 1

A-VECTCR ENTRIES (COEFFICIENT DATA.  FORMAT 9E8.5)

-1.0     C.5      6.0      0.0      20.0     4.C      1.0
2.0      1.C      3C.0     1C.0     1.0      -3.0     1.0      0.0
1.0      0.C      1.0

M-VECTCR ENTRIES (FORMAT 24I3)

  2  1  C  4  1  1  1 -1

OTHER FLAGS AND INPUT DATA.  THESE ARE,  NYQ,OMEGA,DOMEG,
      CMEGF,NBOC,NZRT,DT,T,FMT(1),STEP,LOOP.
      THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
      FCLLCWING FORMAT IF NCT APPLICABLE TC THE PROBLEM
      NOTHING NEEC BE ENTERED.  THE CARDS MUST STILL BE
      INCLLDED.
      1ST CARD      I5,3E10.5,2I5,2E10.5,A6
      2NC CARD      E1C.5,I5
  FOR THIS PROBLEM THE SUPPLEMENTARY DATA ENTRIES ARE,
                    NIL
CATA  ENC
```

```
F1  NC. CF A-VECTOR TERMS                                    N = 20

F2  NO. CF M-VECTOR TERMS                                    N =  8

F3  NO. CF RUNS TC BE MADE.                                  N =  1

F4  ONE CF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                 N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE OF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS CF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER                 N = -0
       SYSTEM BUT NO Z-FORM    N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER CF S                                       N =  1

F7  HIGHEST POWER CF Z                                       N =  1

F8  NUMBER OF VALUES ASSIGNED TO I                           N =  1

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)           N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                         N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                          N = -0

F12  REPORT HEADING OPTION  (N=+,-2)                         N = -2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                               N =  0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS          N = -0

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
        N=DEGREE OF RESULTING Z-FORM DENOMINATOR           N = -0
                   +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                     N = -0

F19  REAL PART  N=-10K                                     N = -0


                        M-VECTOR DATA

FEEDBACK LOOP NO.  1
  SPECIFICATION OF THE FEEDBACK LOOP COMPONENTS
  AS TO DEGREE OF NUMERATOR AND DENOMINATOR

                    UNITY FEEDBACK

FORWARD LOOP NO.  1

    NO. OF TERMS IN NUMERATOR N =  2
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  C

    NO. OF TERMS IN DENOMINATOR  N =  4
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  1  1 -1
```

APPENDIX A.9.3

ROOT LOCUS, BODE, NYQUIST DIAGRAMS

Purpose:

To illustrate the modified Control Systems Analysis program's capability in producing these plots.

Transfer Function:

$$G_{oL} = \frac{30(s+2)}{s(s-3)(s+10)}$$

Outline:

The Root Locus, Bode, and Nyquist diagrams for this system are presented in Schilling(18). The operation of the program was checked by comparing the computed results and those given in the reference. Graphs (2,3,4,5) agree exactly with those given in (18).

For instruction on how to obtain one, or all three of these plots, for a system such as the one above, refer to the listings of the subroutine or subroutines concerned, (A.1,A.2).

These plots can be obtained for linear continuous systems, or for systems with a pure time delay.

The Input Data for this problem is listed on pages 115 to 117.

A problem with a pure time delay element is not included. The problem format would be the same as that of the problem presented in Appendix (A.9.2) except for the additional flags and parameters defined and designated in the listings

of the programs concerned such as NYQ and BODE.

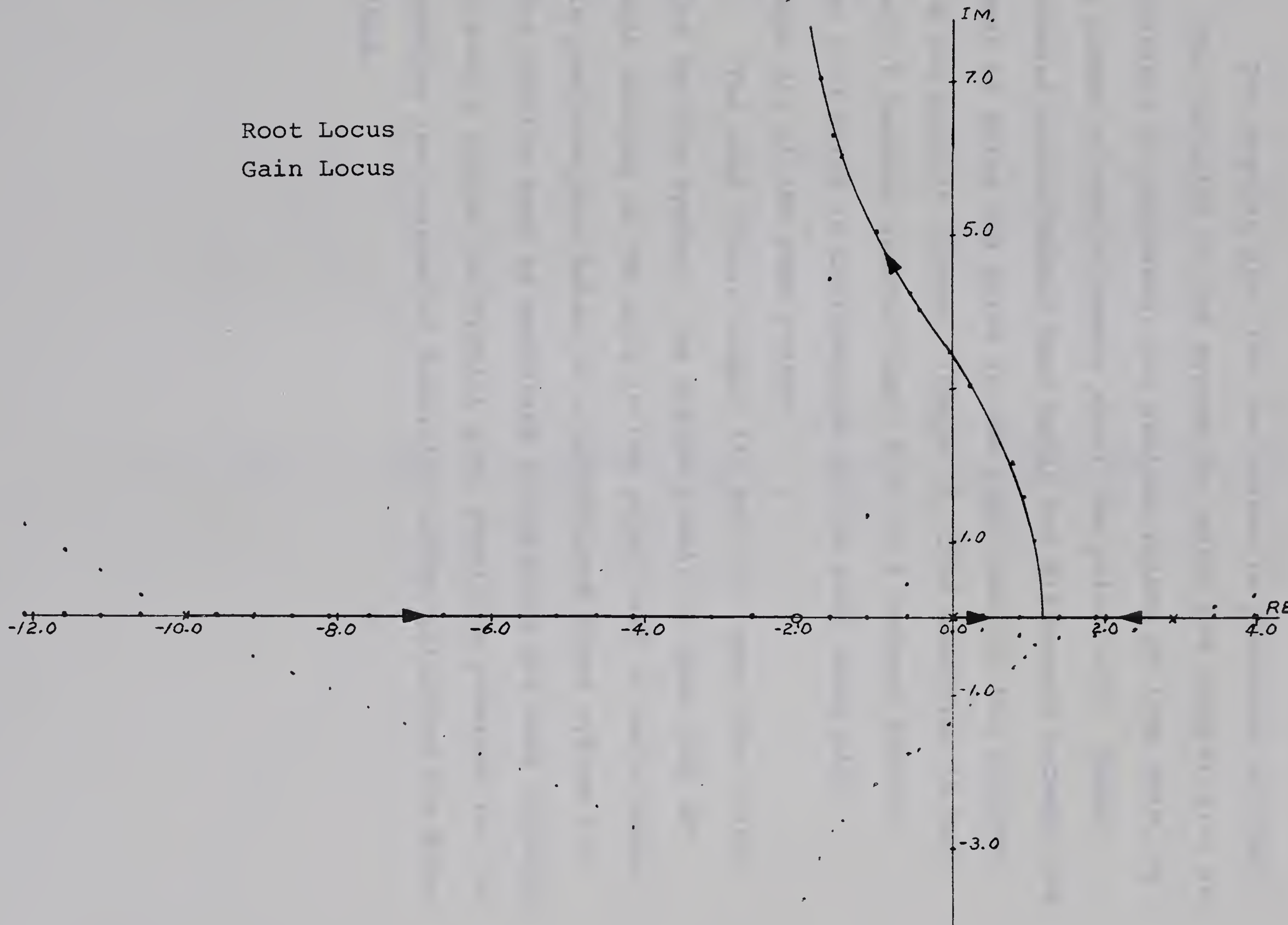
Graph 2 represents the root locus of the system. Two sets of curves are apparent, one composed of black dots, the other a solid line. The solid curve is the root locus diagram and the dotted curve is a plot of the negative value of the loop gain for each point satisfying the characteristic equation. It can be seen, how for poles, the gain curve goes to zero and for zeros it goes off to infinity, and for each branch of the locus there is a corresponding gain branch.

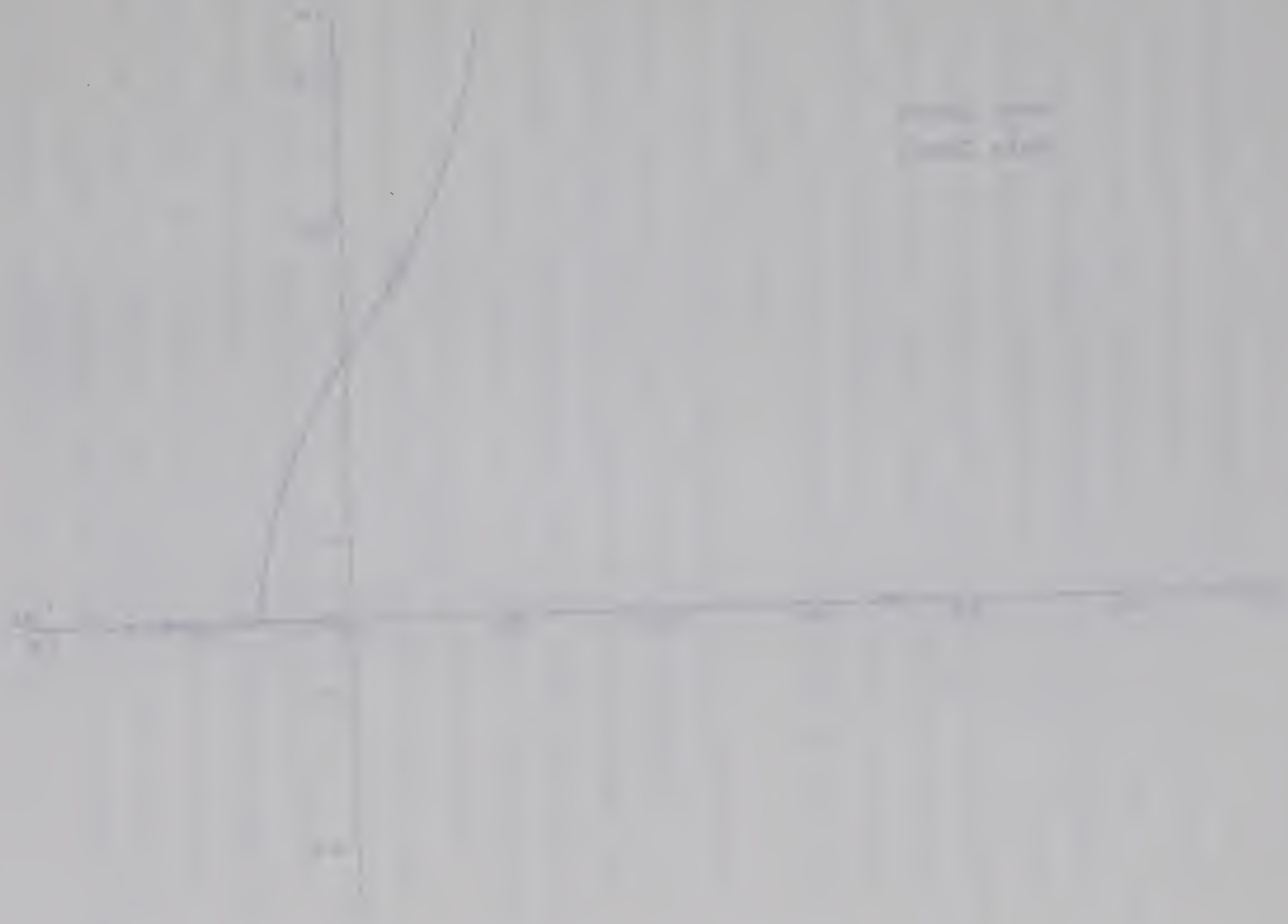
Since the value of the gain plotted is the negative value of the true gain, points on the root locus are indicated by negative values of gain. Points are also calculated for values of true negative gain, as long as the equation $G(s) = -1/K$ is satisfied. These are plotted and written out as positive gain values.

The root locus is defined as the paths of the roots of the characteristic equation as the parameter K is varied from zero to infinity. This excludes all negative values of K, therefore, these points, though satisfying the characteristic equation, must be excluded from the root locus. The user can do this quite easily either through the printed output or by the use of the gain curve plotted with the root locus. When the gain curve has negative values, the points corresponding to these values are on the true root locus.

Graph 2

Root Locus Diagram

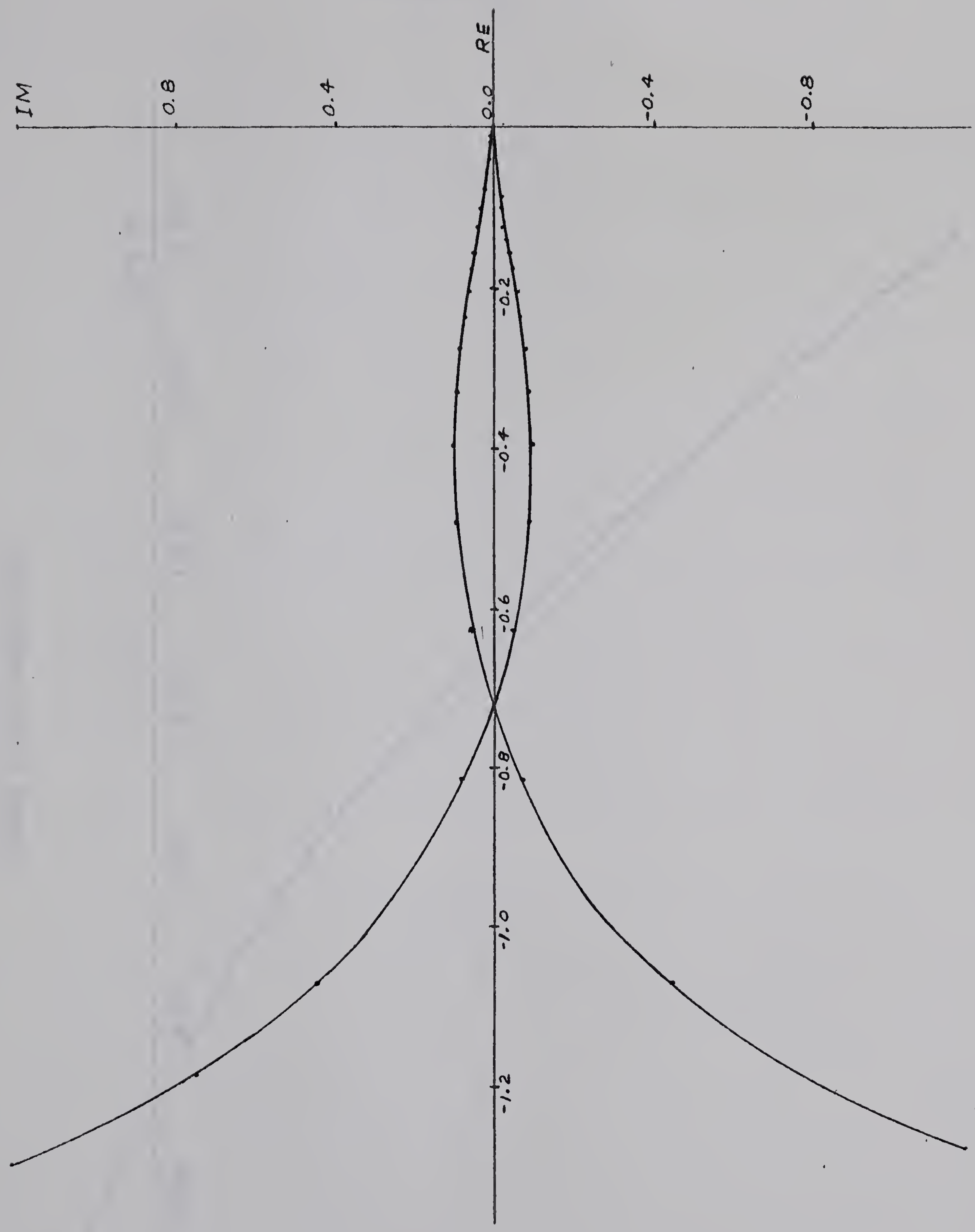




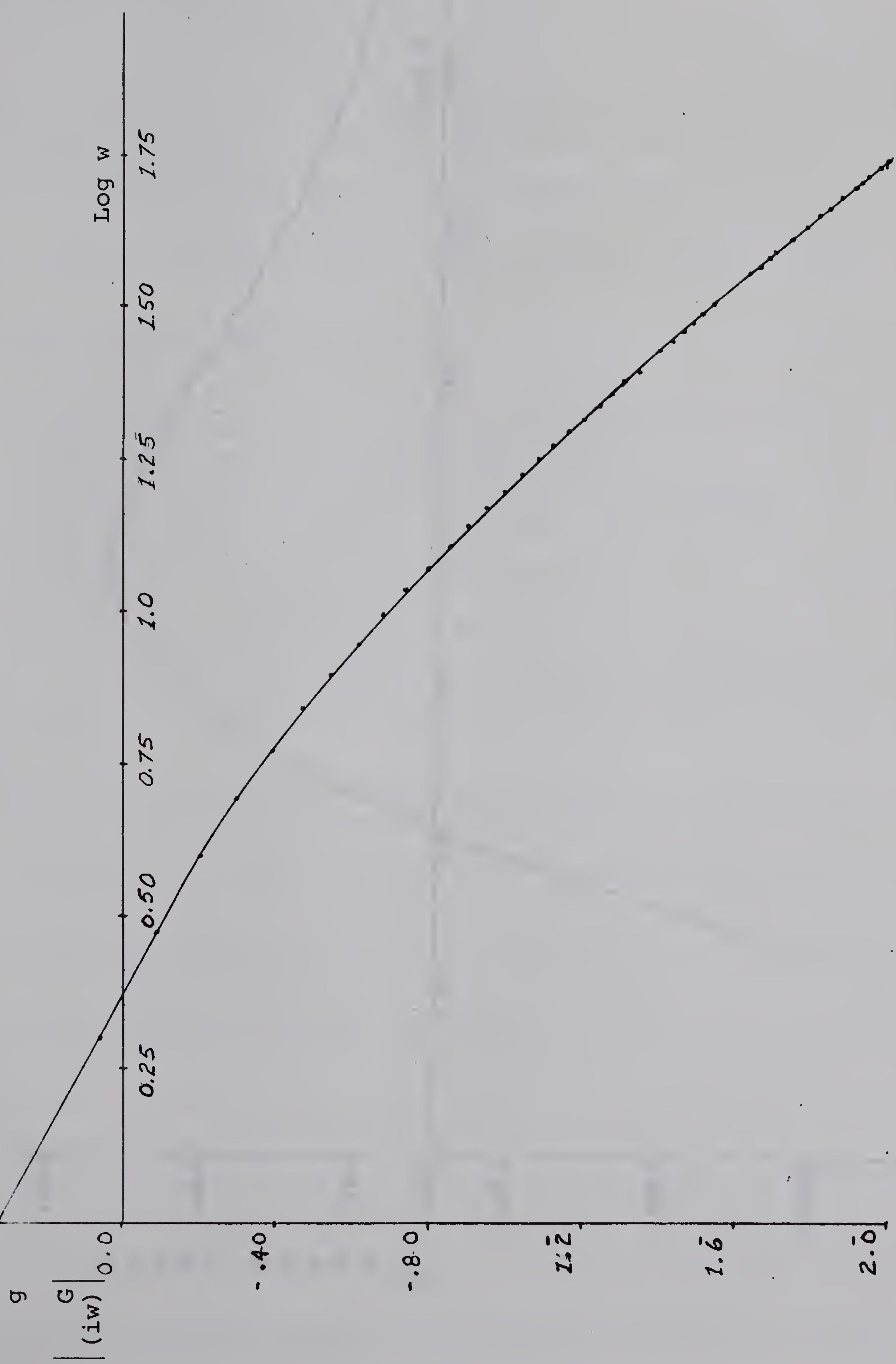
The Nyquist plot for the system is presented as Graph 3. The stability of the system for which the Nyquist plot is calculated is indicated, for various values of loop gain, by the number of encirclements about the point -1/K. These clockwise encirclements must equal the difference between the number of poles and zeros in the right half of the s-plane. From the Nyquist diagram, (Graph 3), this condition is met when 1/K becomes greater than -0.70 or K becomes greater than 1.43 which is corroborated by the root locus plot, (Graph 2), of the same system.

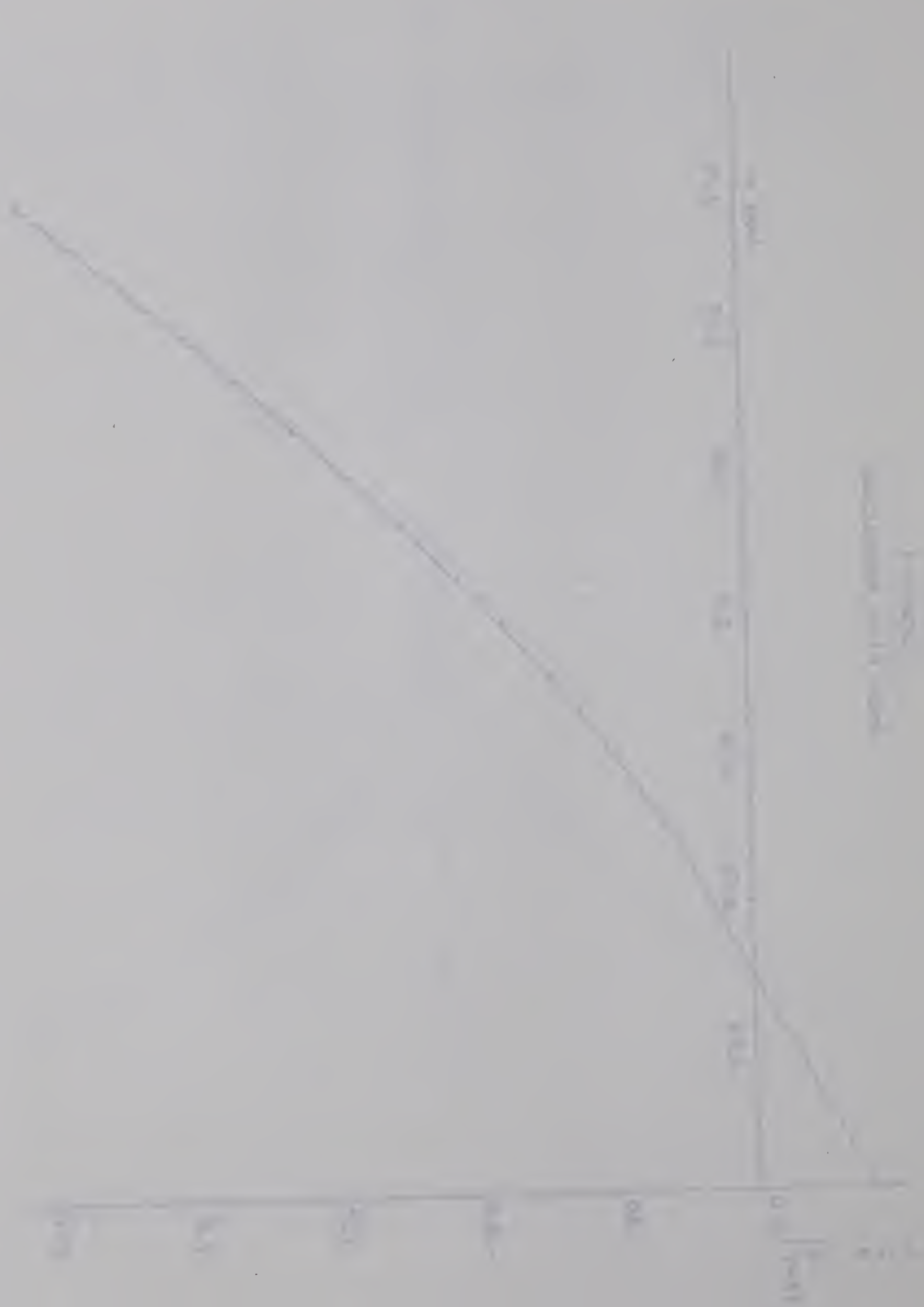
The Bode Plots, Graphs (4) and (5), were also calculated for the system. The system itself is open loop unstable because of the pole in the right half of the s-plane. This positive pole makes it a non-minimum phase system for which caution must be exercised in applying the Bode criteria. For such a system the Nyquist plot should be referred to, to establish the regions of stability before utilizing the Bode method.

Graph 3
Nyquist Plot



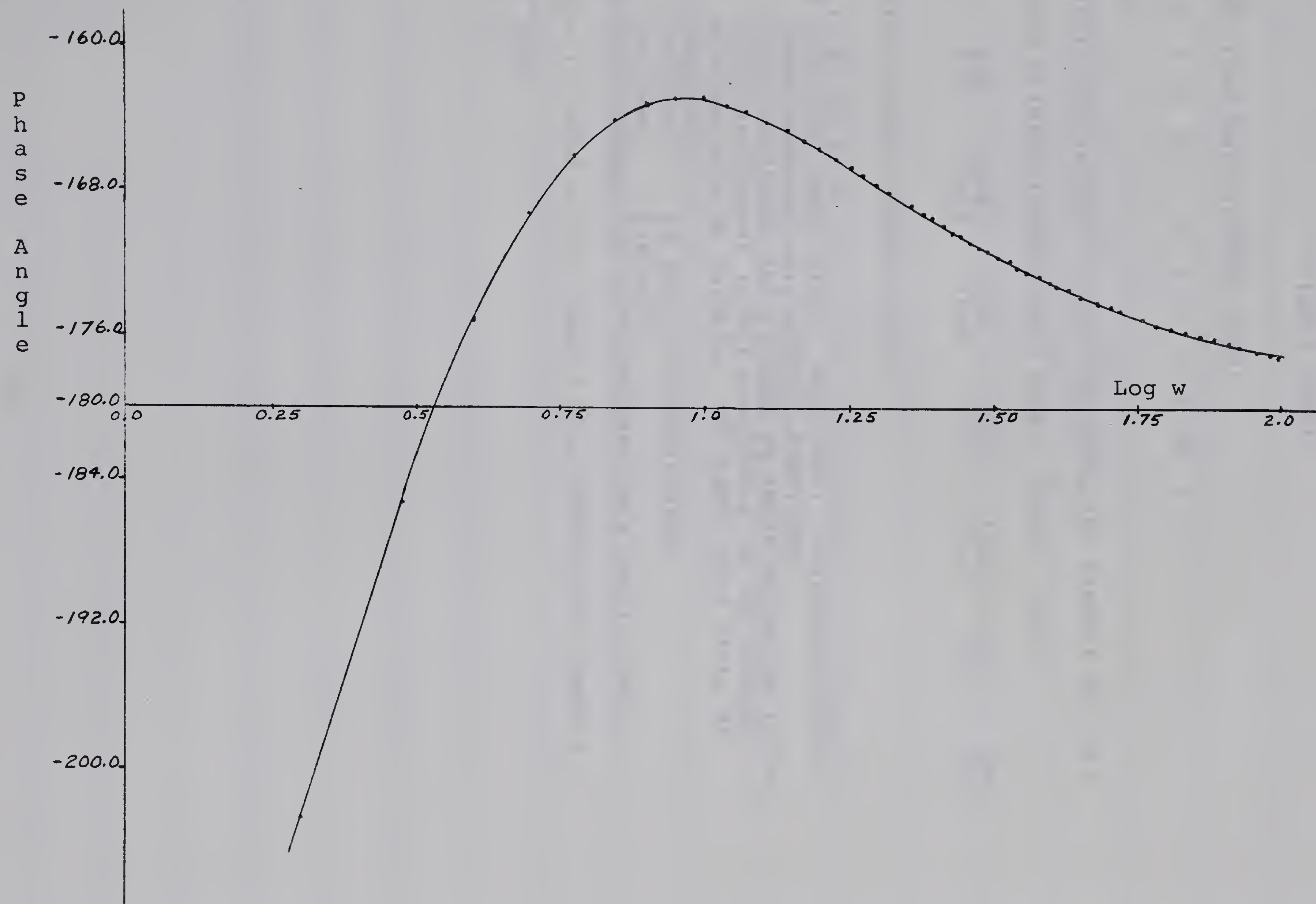
Graph 4
Bode Plot Magnitude

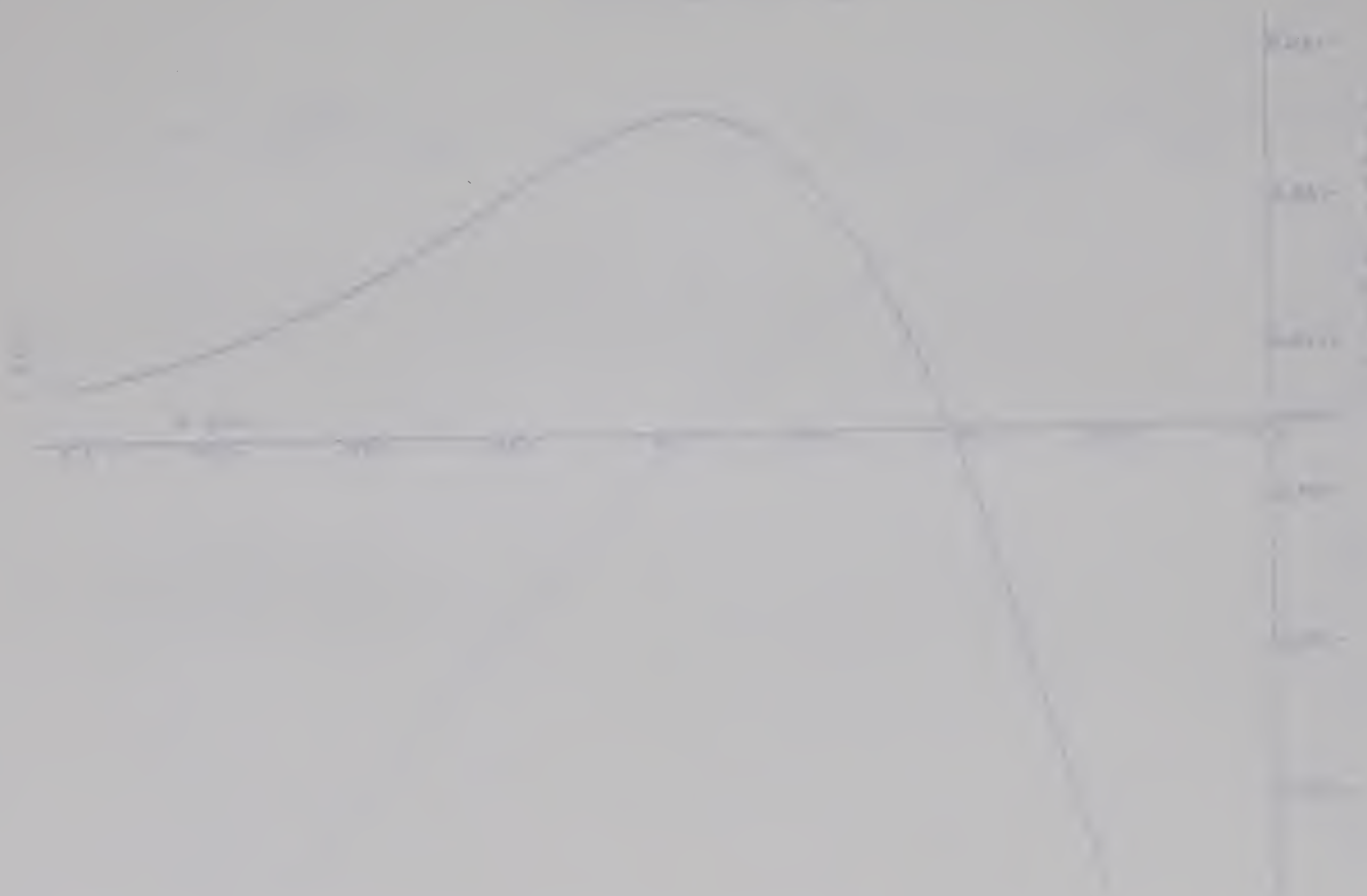




Graph 5

Bode Plot Phase Angle





INPUT DATA

CCNTRCL CARD CATA (FORMAT 24I3)

```
 18  7  1  0  0  1  C  0  1  0  0 -2  0  0
```

TITLE

NYQUIST,BCDE,AND ROCT LOCUS FOR SCHILLING EXAMPLE PG. 214

A-VECTCR ENTRIES (COEFFICIENT DATA. FORMAT 9E8.5)

```
-C.5    1.C     4.0     0.0     12.0    10.0    0.0     0.0
2.C     1.C     30.0    10.0    1.0     -3.0    1.0     0.0
1.0
```

M-VECTCR ENTRIES (FORMAT 24I3)

```
  2  1  0  3  1  1  1
```

OTHER FLAGS AND INPUT DATA. THESE ARE, NYQ,OMEGA,DOMEG,
OMEGF,NBOD,NZRT,DT,T,FMT(1),STEP,LOOP.
THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
FOLLOWING FORMAT IF NOT APPLICABLE TO THE PROBLEM
NOTHING NEED BE ENTERED. THE CARDS MUST STILL BE
INCLUDED.

1ST CARD I5,3E10.5,2I5,2E10.5,A6
2ND CARD E10.5,I5

FOR THIS PROBLEM THE SUPPLEMENTARY DATA ENTRIES ARE,

NYQ = 1, OMEGA = 0.0, DOMEG = 1.0, OMEGF=100.0, NBOD = 1

DATA END

```
F1  NO. OF A-VECTOR TERMS                                     N = 18

F2  NO. OF M-VECTOR TERMS                                     N =  7

F3  NO. OF RUNS TO BE MADE.                                   N =  1

F4  ONE OF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                  N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE OF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS OF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER                  N = -0
       SYSTEM BUT NO Z-FORM   N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER OF S                                        N =  1

F7  HIGHEST POWER OF Z                                        N = -0

F8  NUMBER OF VALUES ASSIGNED TO T                            N = -0

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)            N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                          N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                           N =  0

F12  REPORT HEADING OPTION  (N=+,-2)                          N = -2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                                    N = -0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS               N = -0

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
      N=DEGREE OF RESULTING Z-FORM DENOMINATOR                  N = -0
           +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                          N = -0

F19  REAL PART  N=-10K                                          N = -0


                        M-VECTOR DATA

FEEDBACK LOOP NO.  1
  SPECIFICATION OF THE FEEDBACK LOOP COMPONENTS
  AS TO DEGREE OF NUMERATOR AND DENOMINATOR

                   UNITY FEEDBACK

FORWARD LOOP NO.  1

    NO. OF TERMS IN NUMERATOR N =  2
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
          1  0

    NO. OF TERMS IN DENOMINATOR  N =  3
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
          1  1  1
```

APPENDIX A.9.4

TRANSIENT RESPONSE CALCULATION FOR A A CONTINUOUS SYSTEM (1st order hold)

Purpose:

To illustrate the use of the Z-transform capabilities of the program to predict the transient response of continuous systems to step disturbances in the set point.

Problem Source:

The system was designed to represent a typical control problem. A solution for the response of this system was first obtained using an analogue computer. This was used to check the solution for the same problem given by the digital computer.

Transfer Functions:

Open Loop

$$G_{oL}(s) = \frac{3.6(0.2s+1)}{0.2s(5.0s+1)}$$

Closed Loop

$$G_{cL}(s) = \frac{3.6(0.2s+1)}{s^2 + 0.92s + 3.6}$$

$$= \frac{0.72(s+5.0)}{s^2 + 0.92s + 3.6}$$

Outline:

The analogue solution is calculated for a step input of 1.8. The digital solution is for the same step.

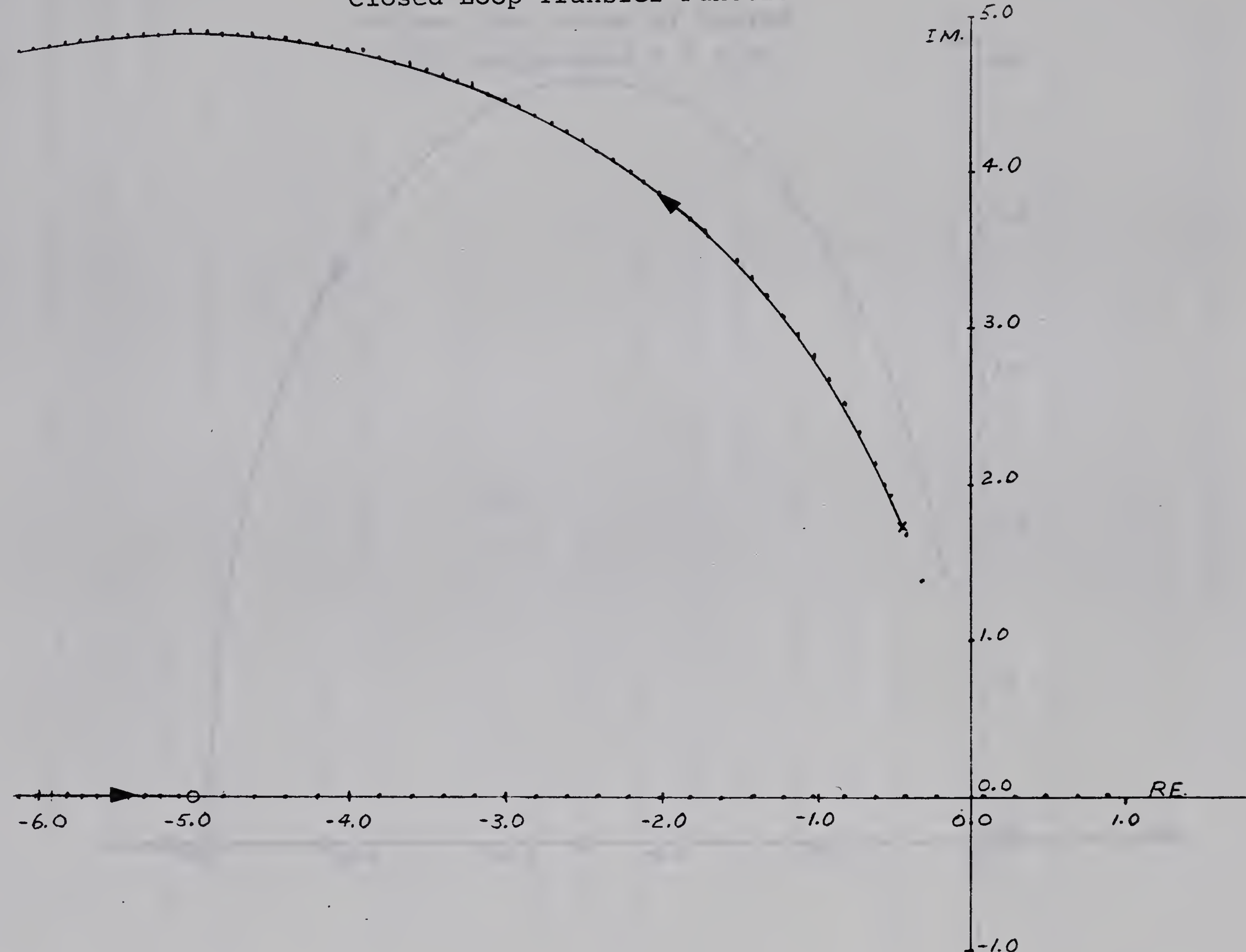
In order to obtain the digital solution a certain procedure must be followed and certain criteria must be met. An outline of the procedure follows; calculations are presented in order to clarify each step.

1. Obtain the closed loop transfer function of the system. This is a user calculation.
2. Using the Control Systems Analysis program determine the root locus for this transfer function, Graph (6).
3. Decide on the type of hold to be used. In this case a 1st order hold is necessary because the order of the denominator is only one greater than the numerator. Combine the hold transfer function and that of the closed loop process.
4. With this hold, pick a sampling period T and calculate the root locus of the modified transfer function, (Graph 7), in the s-plane using the C.S.A. program. If this root locus does not coincide with that of the closed loop transfer function without the hold, a new T (smaller) must be picked and the calculation repeated.

$$G(s) \;=\; e^{sT}\; H_1(s)\; \frac{0.72(s+5.0)}{s^2 + 0.92s + 3.6}$$

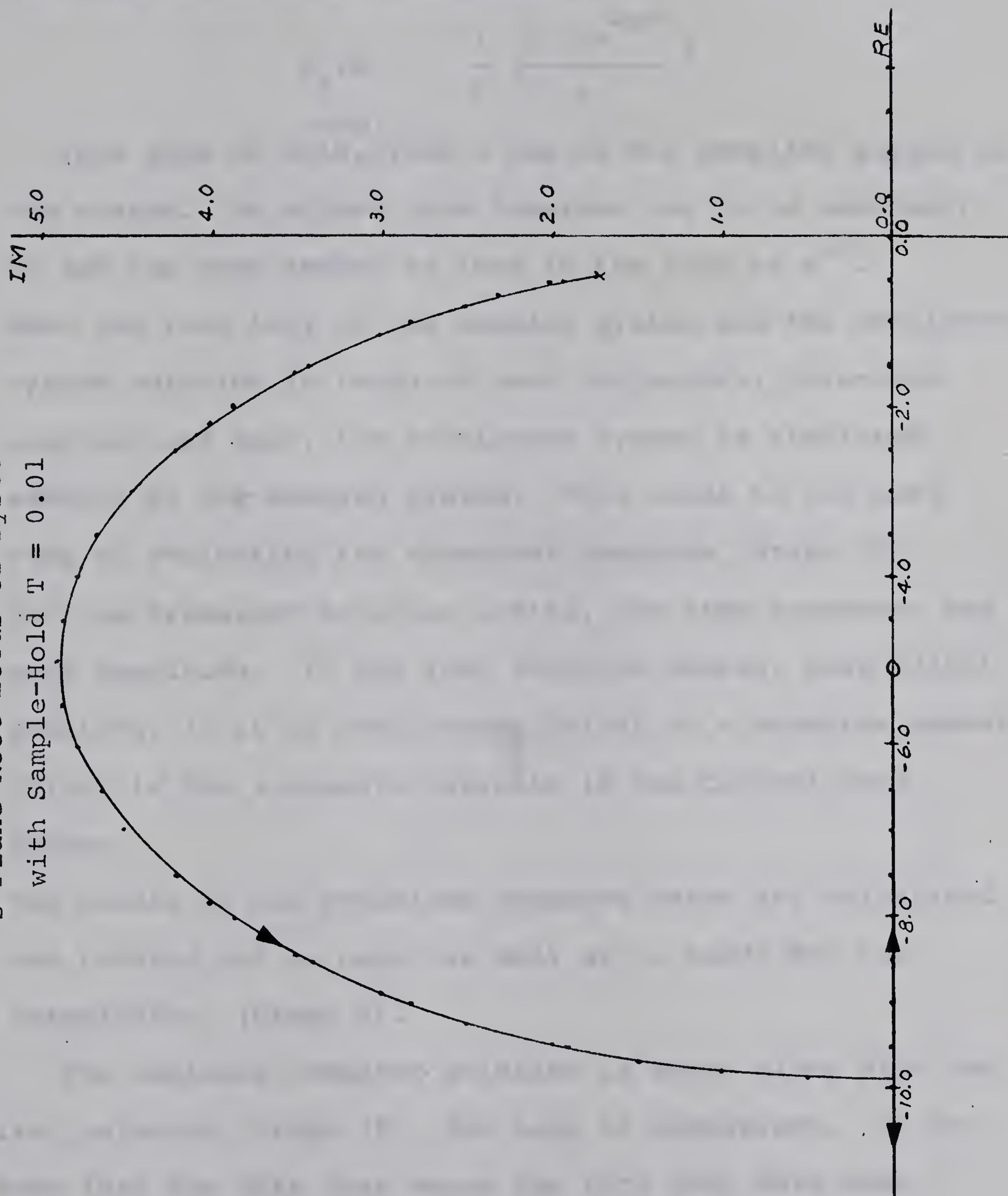
Graph 6

s-Plane Root Locus for The
Closed Loop Transfer Function



Graph 7

s-Plane Root Locus of System
with Sample-Hold T = 0.01



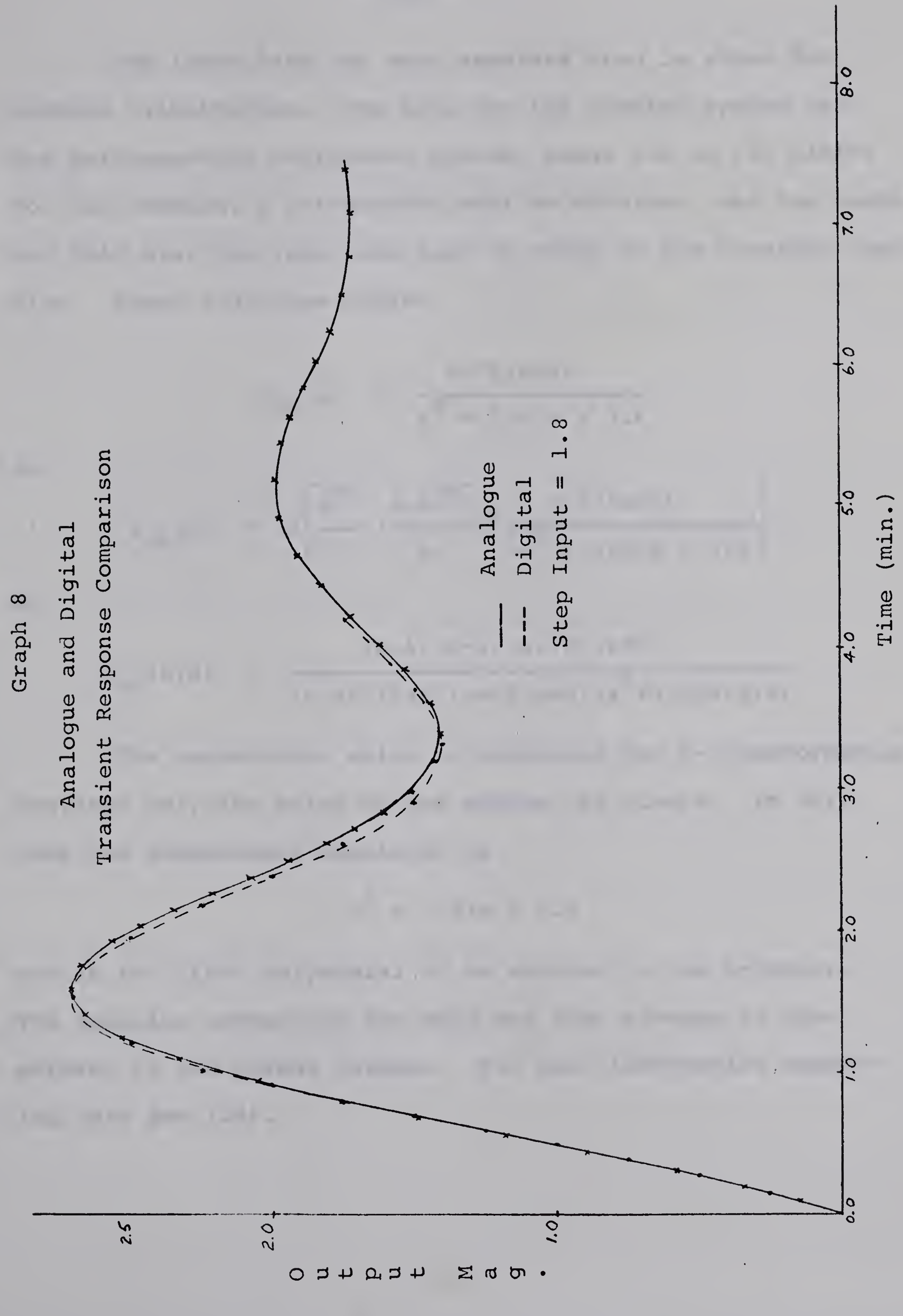
The first order hold is expressed as

$$H_1(s) = \frac{1}{T} \left(\frac{1 - e^{-sT}}{s}\right)^2 .$$

This type of hold usually gives a lag of one sampling period to the system. To offset this inherent lag it is necessary to add the same amount of lead in the form of e^{sT}.

5. When the root loci of the sampled system and the continuous system coincide in terms of root trajectory, pole-zero position and gain, the continuous system is simulated exactly by the sampled system. This leads to the next step of evaluating the transient response, Graph (8).
6. Set the transient solution limits, the time increment and step magnitude. If the root locus is wanted, keep JO(16) positive, if it is not, change JO(16) to a negative number. JO(16) is the sixteenth variable in the Control Card Vector.
7. The points on the transient response curve are calculated and printed out on paper as well as on cards for the Autoplotter, (Graph 8).

The analogue computer solution is shown along with the digital solution, Graph (8), for ease of comparison. It can be seen that for this case where the root loci have been matched almost exactly, T = 0.01, the agreement between the two solutions is excellent.



Graph 8
Analogue and Digital
Transient Response Comparison
—— Analogue
--- Digital
Step Input = 1.8
Time (min.)
Output Mag.
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
1.0
2.0
2.5

The input data for each separate step is shown for problem illustration. The data for the sampled system and the corresponding continuous system, pages 125 to 130 differ for two reasons, a Z-transform must be obtained, and the sample and hold plus the lead term must be added to the transfer function. These additions convert

$$G_{cL}(s) = \frac{0.72(s+5)}{s^2 + 0.92s + 3.6}$$

to

$$G_{cL}(s) = Z\left[\frac{e^{sT}}{T}\left(\frac{1-e^{-sT}}{s}\right)^2\left(\frac{0.72(s+5)}{s^2 + 0.92s + 3.6}\right)\right]$$

or

$$G_{cL}(s,Z) = \frac{(Z-1)(Z-1)(0.72)(s+5)}{(0.01)(Z+0)(s+0)(s+0)(s^2+0.92s+3.6)}$$

The denominator which is submitted for Z-transformation contains only the poles of the system, no others. In this case the denominator submitted is

$$s^2 + 0.92s + 3.6$$

and is the first polynomial to be encoded in the A-vector. The equation containing the hold and time advance is then entered in the normal fashion. For more information regarding this see (24).

INPUT DATA

```
CCNTRCL CARC CATA (FCRMAT 24I3)

 15  5  1  C  0  2  C  C  1  0  0 -2  0  0  0  0

TITLE
 RCOT LCCUS OF THE CLOSED LCOP (CONTINUOUS)

A-VECTCR ENTRIES (COEFFICIENT DATA.  FORMAT 9E8.5)

-0.1     1.C      1.0      0.0      6.0      5.C      0.0      0.0
5.0      1.C      C.72     3.6      0.92     1.C

M-VECTCR ENTRIES (FORMAT 24I3)

  2  1  C  1  2

CTHER FLAGS ANC INPUT DATA.  THESE ARE,  NYQ,OMEGA,DOMEG,
      CMEGF,NBOD,NZRT,DT,T,FMT(1),STEP,LCOP.
      THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
      FCLLOWING FORMAT IF NOT APPLICABLE TC THE PROBLEM
      NOTHING NEEC BE ENTERED.  THE CARDS MUST STILL BE
      1ST CARD       I5,3E10.5,2I5,2E10.5,A6
      2NC CARD       E1C.5,I5
  FOR THIS PROBLEM THE SUPPLEMENTARY CATA ENTRIES ARE,

                     NIL

CATA  ENC
```

```
F1  NC. CF A-VECTCR TERMS                                  N = 15

F2  NO. CF M-VECTOR TERMS                                  N =  5

F3  NO. CF RUNS TO BE MADE.                                N =  1

F4  ONE CF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD               N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE CF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS CF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER               N =  0
       SYSTEM BUT NO Z-FORM   N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER OF S                                     N =  2

F7  HIGHEST POWER OF Z                                     N =  0

F8  NUMBER OF VALUES ASSIGNED TO T                         N =  0

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)         N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                       N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                        N =  0

F12 REPORT HEADING OPTION  (N=+,-2)                        N = -2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                                N =  0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS           N = -0

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
       N=DEGREE OF RESULTING Z-FORM DENOMINATOR              N =  0
                  +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                      N = -0

F19  REAL PART  N=-10K                                      N = -0
```

M-VECTOR DATA

```
FEEDBACK LOOP NO.  1
  SPECIFICATION OF THE FEEDBACK LOOP COMPONENTS
  AS TO DEGREE OF NUMERATOR AND DENOMINATOR

              UNITY FEEDBACK

FORWARD LOOP NO.  1

    NO. OF TERMS IN NUMERATOR N =  2
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  0

    NO. OF TERMS IN DENOMINATOR  N =  1
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            2
```

INPUT DATA

```
CONTROL CARD DATA (FORMAT 24I3)

 29 14  1  0-11  2  5  1  1  0  0  2  0150  0  2

TITLE

 TRANSIENT RESPONSE. 1ST ORDER HOLD. SAMPLE TIME =0.01 MIN.

A-VECTOR ENTRIES (COEFFICIENT DATA.  FORMAT 9E8.5)

-0.5     0.5      2.0      0.0      10.0     10.0     0.01     0.0
 3.6     0.92     1.0      5.0      1.0      0.722   -1.0      1.0
-1.0     1.0      3.6      0.92     1.0      0.0      1.0      0.0
 1.0     0.0      1.0      0.01

M-VECTOR ENTRIES (FORMAT 24I3)

  1  2  0  4  1  0 -1 -1  5  2  1  1 -1  0

OTHER FLAGS AND INPUT DATA.  THESE ARE,  NYQ,OMEGA,DOMEG,
      OMEGF,NBOD,NZRT,DT,T,FMT(1),STEP,LOOP.
      THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
      FOLLOWING FORMAT IF NOT APPLICABLE TO THE PROBLEM
      NOTHING NEED BE ENTERED.  THE CARDS MUST STILL BE
      INCLUDED.
      1ST CARD      I5,3E10.5,2I5,2E10.5,A6
      2ND CARD      E10.5,I5
  FOR THIS PROBLEM THE SUPPLEMENTARY DATA ENTRIES ARE,

  DT = 0.01, T = 4.5, FMT(1) = MIN., STEP = 1.8

DATA  END
```

```
F1  NO. OF A-VECTOR TERMS                                      N = 29

F2  NO. OF M-VECTOR TERMS                                      N = 14

F3  NO. OF RUNS TO BE MADE.                                    N =  1

F4  ONE OF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                   N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE OF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS OF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER                   N =-11
       SYSTEM BUT NO Z-FORM   N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER OF S                                         N =  2

F7  HIGHEST POWER OF Z                                         N =  5

F8  NUMBER OF VALUES ASSIGNED TO T                             N =  1

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)             N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                           N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                            N =  0

F12 REPORT HEADING OPTION  (N=+,-2)                            N =  2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                                   N =   0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS              N =150

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
       N=DEGREE OF RESULTING Z-FORM DENOMINATOR                N =   2
                  +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                         N =  -1

F19  REAL PART  N=-10K                                         N =  -0


                    M-VECTOR DATA

FEEDBACK LOOP NO.  1
  SPECIFICATION OF THE FEEDBACK LOOP COMPONENTS
  AS TO DEGREE OF NUMERATOR AND DENOMINATOR

                 UNITY FEEDBACK

FORWARD LOOP NO.  1

    NO. OF TERMS IN NUMERATOR N =  4
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  0 -1 -1

    NO. OF TERMS IN DENOMINATOR  N =  5
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            2  1  1 -1  0
```

APPENDIX A.9.5

TRANSIENT RESPONSE CALCULATION FOR A CONTINUOUS SYSTEM (3^{rd} order, zero order hold)

Purpose:

To further illustrate the procedure to be followed in calculating the transient response of a system to a step input using the Z-transform.

Problem Source:

This is essentially the same problem as Problem A.9.4.1, however, a first order capacity has been given to the control element, where previously the control element was represented as a pure gain.

Block Diagram Representing the Problem:

This is the same as in A.9.4, the change is that

$$G_v(s) = \frac{K_2 R_v}{\tau_o s+1}$$

Parameter Values

$$\tau_p = 5.0$$

$$K_1 = 1.2$$

$$K_2 = 1.5$$

$$\tau_i = 0.2$$

$$\tau_p = 1.0$$

Transfer Function:

Open Loop

$$G_{oL}(s) = \frac{3.6(0.2s+1)}{0.2s(s+1)(5.0s+1)}$$

Outline:

A root locus analysis of the system containing the integral time $\tau_i = 0.2$ showed that the system would be unstable even for very low loop gains. This root locus is not reproduced herein, but by looking at the transfer function, it can be seen that poles will occur at 0.0, -1, and 0.2 and a zero will occur at -5.0. One root started at the -1.0 pole and went to the zero at -5.0 and the other two roots met and branched between -0.2 and 0.0. Their paths then crossed the imaginary axis.

By moving the controller zero from -5.0 to -0.1 the system can be stabilized, the roots starting at -0.2 and -1.0 now meet and split into the imaginary plane and continue in a vertical line to plus and minus infinity. The system is now indicated stable by the root locus, (not reproduced), for all loop gains. Changing the controller zero from -5.0 to -0.1 is equivalent to changing the integral time from 0.2 to 10 minutes so that now the system transfer function is

$$G_{oL} = \frac{3.6(10s+1)}{10s(s+1)(5.0s+1)}$$

The user must now calculate the closed loop transfer function. Thus, for unity feedback

$$G_{cL}(s) = \frac{3.6(10s+1)}{50s^3 + 60s^2 + 46.0s + 3.6}$$

The Control Systems Analysis program cannot transform s-polynomials of order greater than two to their Z-transform counterparts, therefore, it is necessary to factor the denominator. This was done using the Share library program(23) and the factors obtained were 50s + 4.3752 and s^2 + 1.1125s + 0.8226. Thus, the closed loop transfer function becomes

$$G_{cL}(s) = \frac{3.6(10s+1)}{(50s+4.3767)(s^2+1.1125s+0.8226)}$$

Following the procedure outlined in Appendix (A.9.4) the root locus of the above closed loop transfer function is calculated and plotted to use as a future comparison, (Graph 9).

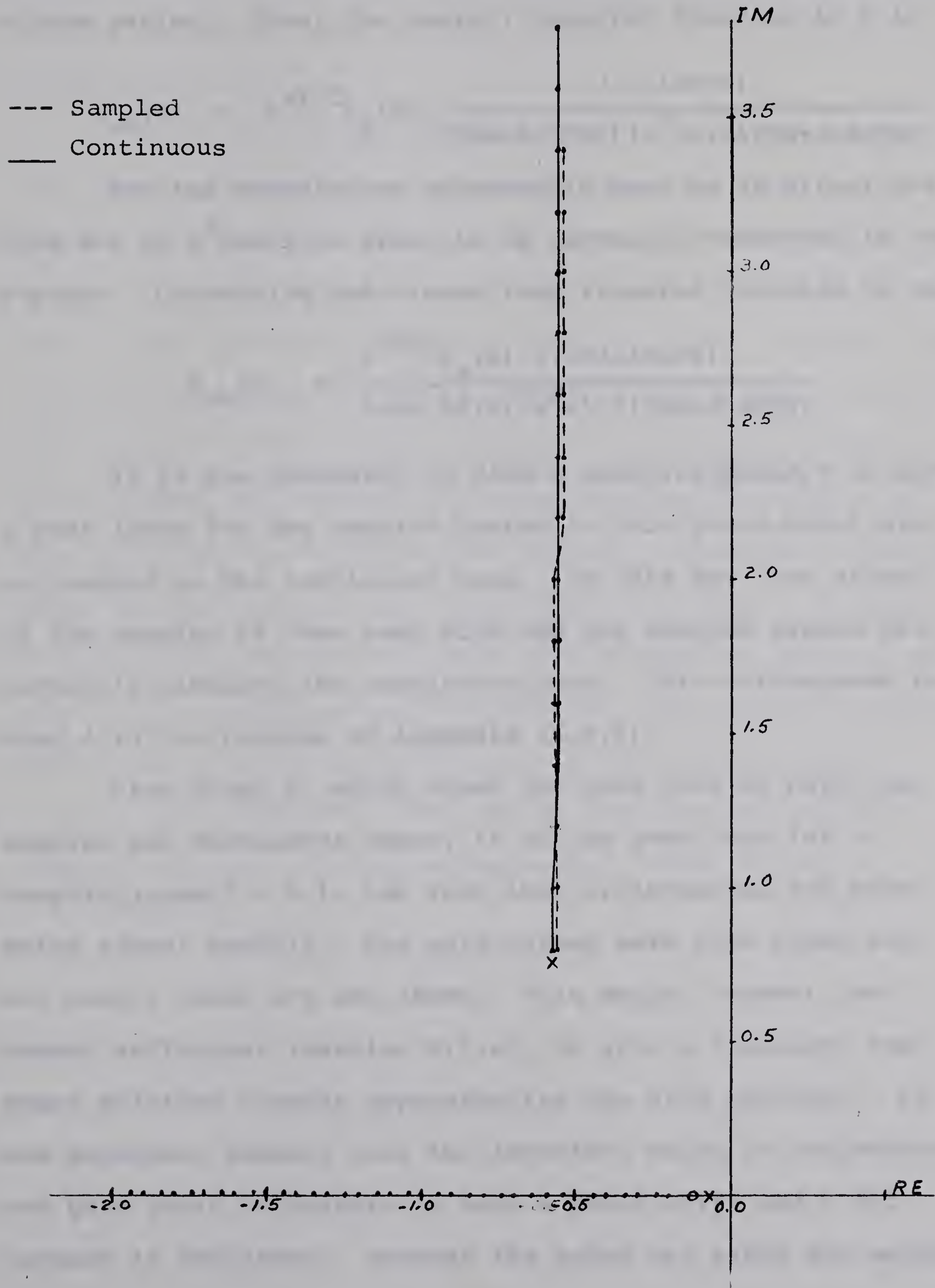
Since the order of the denominator of the s-transfer function is now two less than that of the numerator it is now possible to use the zero order hold as the sample. This sampler is of the form

$$H_o(s) = \frac{1 - e^{-sT}}{s}$$

and is simpler than the first order hold. With the zero order hold a built in lag of half a time period is usually present. In

Graph 9

Root Loci of Continuous and Sampled Systems



order to account for this it is necessary to add a lead of half a time period. Thus, the overall transfer function in s is

$$G_{cL}(s) = e^{sT/2}H_o(s)\ \frac{3.6(10s+1)}{(50s+4.3762)(s^2+1.1125s+0.8226)}$$

Now the denominator polynomials must be in either the form s+b or s^2+as+b in order to be correctly converted to the Z-plane. Converting the closed loop transfer function we get

$$G_{cL}(s) = \frac{e^{sT/2}H_o(s)\ 0.072(10s+1)}{(s+0.0875)(s^2+1.1125s+0.8226)}$$

It is now necessary to find a sampling period, T to give a root locus for the sampled system in this plane which will correspond to the continuous case. In this way, the effect of the sampler is done away with and the sampled system will correctly simulate the continuous case. This corresponds to step 4 in the problem of Appendix (A.9.4).

From Graph 9, which shows the root loci of both the sampled and continuous cases, it can be seen that for a sampling period T = 0.1, the root loci trajectories and poles match almost exactly. The gain curves were also close but not exact; these are not shown. This match, however, was deemed sufficient (Section 8.5.a), to give a transient response solution closely approximating the true solution. It was expected, though, that the imperfect match in trajectory and gain would contribute to both a phase shift and a difference in amplitude. Because the poles and zeros are matched,

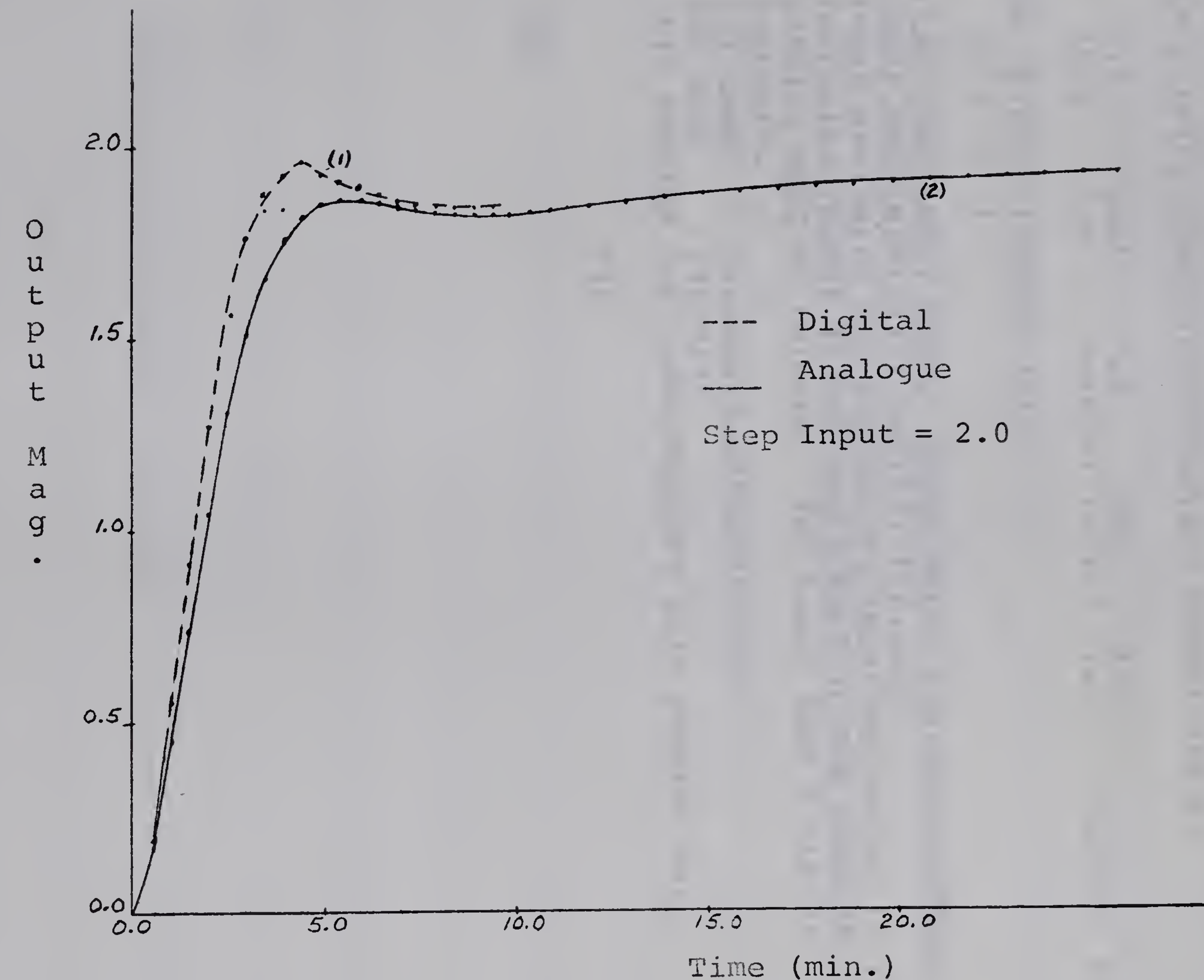
the form of the response or the oscillatory character of the digital solution, should be the same as that for the continuous solution.

Steps 6 and 7, (Appendix A.9.4), were then carried out so that the transient curve no. (1) graph (10) was obtained. The analogue solution is shown as curve no. (2) graph (10). The expected result was obtained. There is present a small shift both in phase and amplitude, however, the oscillatory response is quite similar. For process control work, where the oscillatory response is the main concern to the designer, an error of this magnitude can be tolerated. This error may be eliminated by smaller T, however, round-off error and computation time may become excessive. It should be noted that when a zero order hold is used, a lead of less than a full sample period is employed. This makes it necessary to put a 1 in position JO(11) of the control card.

The input data used for this problem is listed on pages 137 to 143. The disturbance was a step = 2.0.

Graph 10

Transient Response Curves



```
                         INPUT  DATA

CONTROL CARD DATA (FORMAT 24I3)

 17  6  1  0  0  2  0  0  1  0  0 -2  0  0  0  0

TITLE

 ROOT LOCUS OF THE CLOSED LOOP (CONTINUOUS)

A-VECTOR ENTRIES (COEFFICIENT DATA.  FORMAT 9E8.5)

-0.02   0.2     0.01    0.0     2.0     4.0
1.0     10.0    3.6     0.82263 1.1125  1.0     4.3762  50.0

M-VECTOR ENTRIES (FORMAT 24I3)
  2  1  0  2  2  1

OTHER FLAGS AND INPUT DATA.  THESE ARE,  NYQ,OMEGA,DOMEG,
      OMEGF,NBOD,NZRT,DT,T,FMT(1),STEP,LOOP.
      THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
      FOLLOWING FORMAT IF NOT APPLICABLE TO THE PROBLEM
      NOTHING NEED BE ENTERED.  THE CARDS MUST STILL BE
      INCLUDED.
      1ST CARD     I5,3E10.5,2I5,2E10.5,A6
      2ND CARD     E10.5,I5
  FOR THIS PROBLEM THE SUPPLEMENTARY DATA ENTRIES ARE,

                         NIL

DATA  END
```

```
F1   NO. OF A-VECTOR TERMS                                  N = 17

F2   NO. OF M-VECTOR TERMS                                  N =  6

F3   NO. OF RUNS TO BE MADE.                                N =  1

F4   ONE OF FOUR OPTIONS SPECIFIED
       -IF Z-TRANSFORM TO BE COMPUTED  N=0
       -TWO SAMPLER SYSTEM. N=N
       -ALL TRANSFER FUNCTIONS IN ONE FORWARD               N =  0
        LOOP WITH UNITY FEEDBACK.  N=0
       -OTHERWISE  N=N

F5   ONE OF THREE OPTIONS SPECIFIED.
       -FOR ROOT LOCUS OF CONTINUOUS SYSTEM OR
        FOR A SYSTEM IN Z-FORM.  N=0
       -FOR ROOT LOCI POINTS OF A ONE-SAMPLER               N =  0
        SYSTEM BUT NO Z-FORM    N=1
       -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
        POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6   HIGHEST POWER OF S                                     N =  2

F7   HIGHEST POWER OF Z                                     N =  0

F8   NUMBER OF VALUES ASSIGNED TO T                         N =  0

F9   ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)         N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                        N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                         N =  0

F12  REPORT HEADING OPTION  (N=+,-2)                        N = -2
       UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                                    N =  0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS               N =  0

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
       N=DEGREE OF RESULTING Z-FORM DENOMINATOR                 N =  0
                  +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                          N = -0

F19  REAL PART  N=-10K                                          N = -0


                         M-VECTOR DATA

FEEDBACK LOOP NO.  1
  SPECIFICATION OF THE FEEDBACK LOOP COMPONENTS
  AS TO DEGREE OF NUMERATOR AND DENOMINATOR

                    UNITY FEEDBACK

FORWARD LOOP NO.  1

    NO. OF TERMS IN NUMERATOR N =  2
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  0

    NO. OF TERMS IN DENOMINATOR  N =  2
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            2  1
```

INPUT DATA

```
CONTROL CARD DATA (FORMAT 24I3)

 30 14  1  0-10  3  5  2  1  0  1 -2  0 50  0  3

TITLE

 TRANSIENT RESPONSE.  ZEROE ORDER HOLD.

A-VECTOR ENTRIES (COEFFICIENT DATA.  FORMAT 9E8.5)

-0.05   0.2     1.0     0.0     2.0     4.0     0.1     0.05
0.82263 1.1125  1.0     0.0875  1.0     1.0     10.0    .072
-1.0    1.0     0.0     1.0     0.82263 1.1125  1.0
0.0875  1.0     0.0     1.0     0.0     1.0

M-VECTOR ENTRIES (FORMAT 24I3)

  2  2  1  0  4  1  0 -1-21  4  2  1  1 -1

OTHER FLAGS AND INPUT DATA.  THESE ARE,  NYQ,OMEGA,DOMEG,
      OMEGF,NBOD,NZRT,DT,T,FMT(1),STEP,LOOP.
      THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
      FOLLOWING FORMAT IF NOT APPLICABLE TO THE PROBLEM
      NOTHING NEED BE ENTERED.  THE CARDS MUST STILL BE
      INCLUDED.
      1ST CARD      I5,3E10.5,2I5,2E10.5,A6
      2ND CARD      E10.5,I5
  FOR THIS PROBLEM THE SUPPLEMENTARY DATA ENTRIES ARE,

  DT=0.1,  T=10.0,  FMT(1)=MIN.,  STEP=2.0

DATA  END
```

```
F1  NO. OF A-VECTOR TERMS                                    N = 30

F2  NO. OF M-VECTOR TERMS                                    N = 14

F3  NO. OF RUNS TO BE MADE.                                  N =  1

F4  ONE OF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                 N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE OF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS OF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER                 N =-10
       SYSTEM BUT NO Z-FORM   N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER OF S                                       N =  3

F7  HIGHEST POWER OF Z                                       N =  5

F8  NUMBER OF VALUES ASSIGNED TO T                           N =  2

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)           N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                         N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                          N =  1

F12  REPORT HEADING OPTION  (N=+,-2)                         N = -2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTICN,USUALLY N=0                               N =  0

F14  TERMS IN SERIES FCR G*(S),N=0 GIVES 19 TERMS          N = 50

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
      N=DEGREE OF RESULTING Z-FORM DENOMINATCR             N =  3
                  +N  RCOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                     N = -0

F19  REAL PART  N=-10K                                     N = -0

                  M-VECTOR DATA

                  NO FEEDBACK

FORWARD LCOP NC.  1

    NO. CF TERMS IN NUMERATOR N =  4
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  0 -1-21

    NO. CF TERMS IN DENOMINATOR  N =  4
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            2  1  1 -1
```

APPENDIX A.9.6

TRANSIENT RESPONSE OF A SAMPLED-DATA SYSTEM

DIGITAL CONTROLLER DESIGN

Purpose:

To illustrate the procedure followed in obtaining the response of a digitally controlled system to a step change in set point, and, in the design of a digital controller using the Z-plane.

Transfer Function:

$$H_o(s)G_p(s) = \frac{1 - e^{-sT}}{s} \; \frac{0.5}{(s+1)(5s+1)} \qquad (1)$$

Outline: First Example:

1. Take the Z-transform and find the Z-plane root locus for transfer function (1). To do this a sampling period must be decided on. For this problem a sampling period of one minute was chosen. The computed Z-transform is

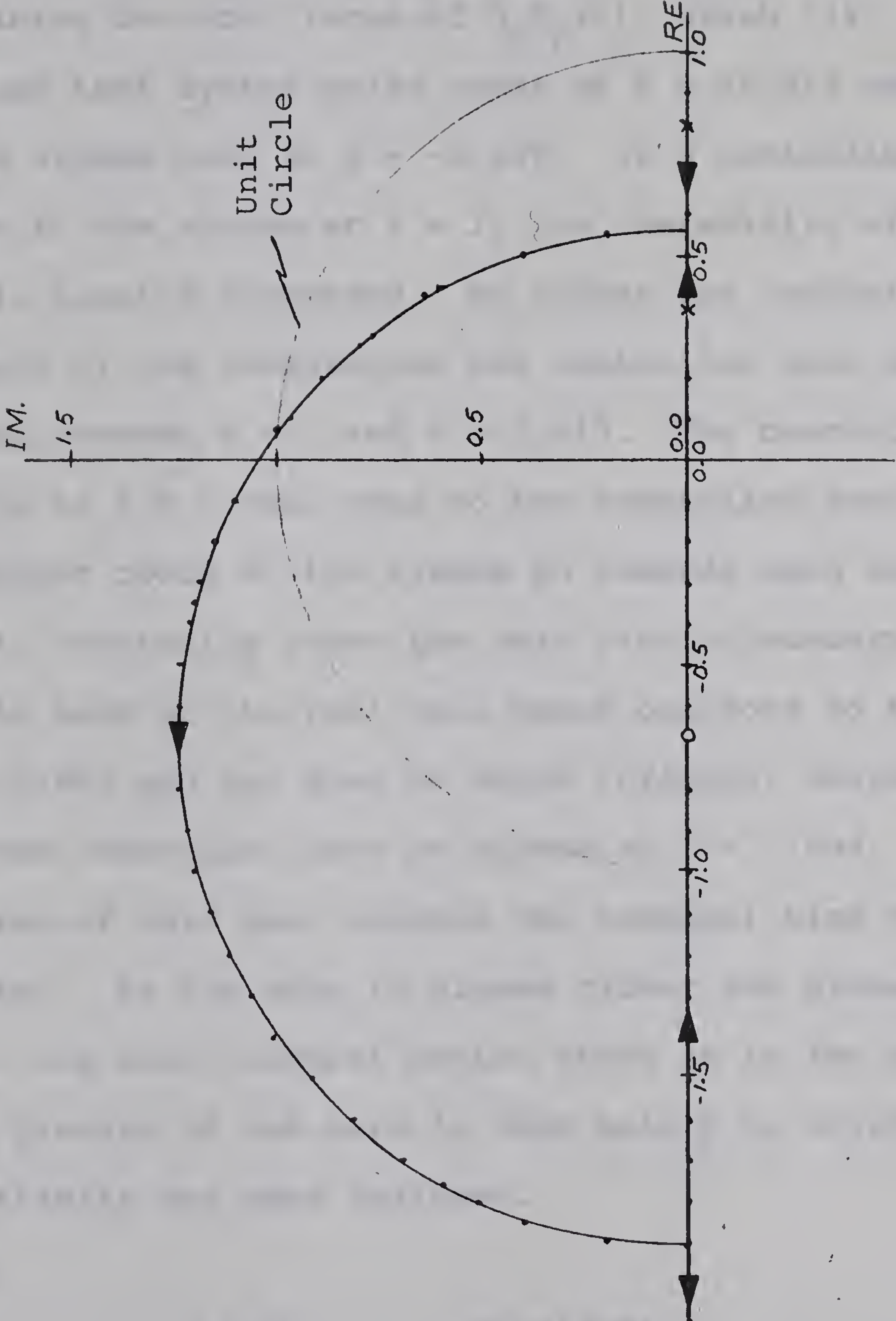
$$(H_oG_p)(Z) = \frac{0.03428Z + 0.023014}{Z^2 - 1.1866Z + 0.301194}$$

   The root locus was also computed for $H_oG_p(Z)$ in the Z-plane. This is shown in Graph (11).

2. Using this root locus a controller can be specified in the Z-plane which will control the system. In order to obtain zero offset at steady-state, a pole must be

Graph 11

Z-Plane Root Locus of Process and Hold



specified at Z = 1. Zeros must not exceed the poles in the controller, therefore, one zero can be specified. Examining the root locus of $H_oG_p(Z)$, Graph (11), it can be seen that system poles occur at Z = +0.819 and Z = 0.368 and a system zero at Z = -0.669. If a controller pole is added to the system at Z = 1, the instability of the system is greatly increased. To offset the destabilizing influence of the integration the controller zero must be placed between Z = 1 and Z = 0.819. The controller root starts at Z = 1 then goes to the controller zero. The two other roots of the system go towards each other, meet, split, eventually cross the unit circle boundary, then circle back to the real axis where one goes to the zero at Z = -0.669 and one goes to minus infinity, Graph (12).

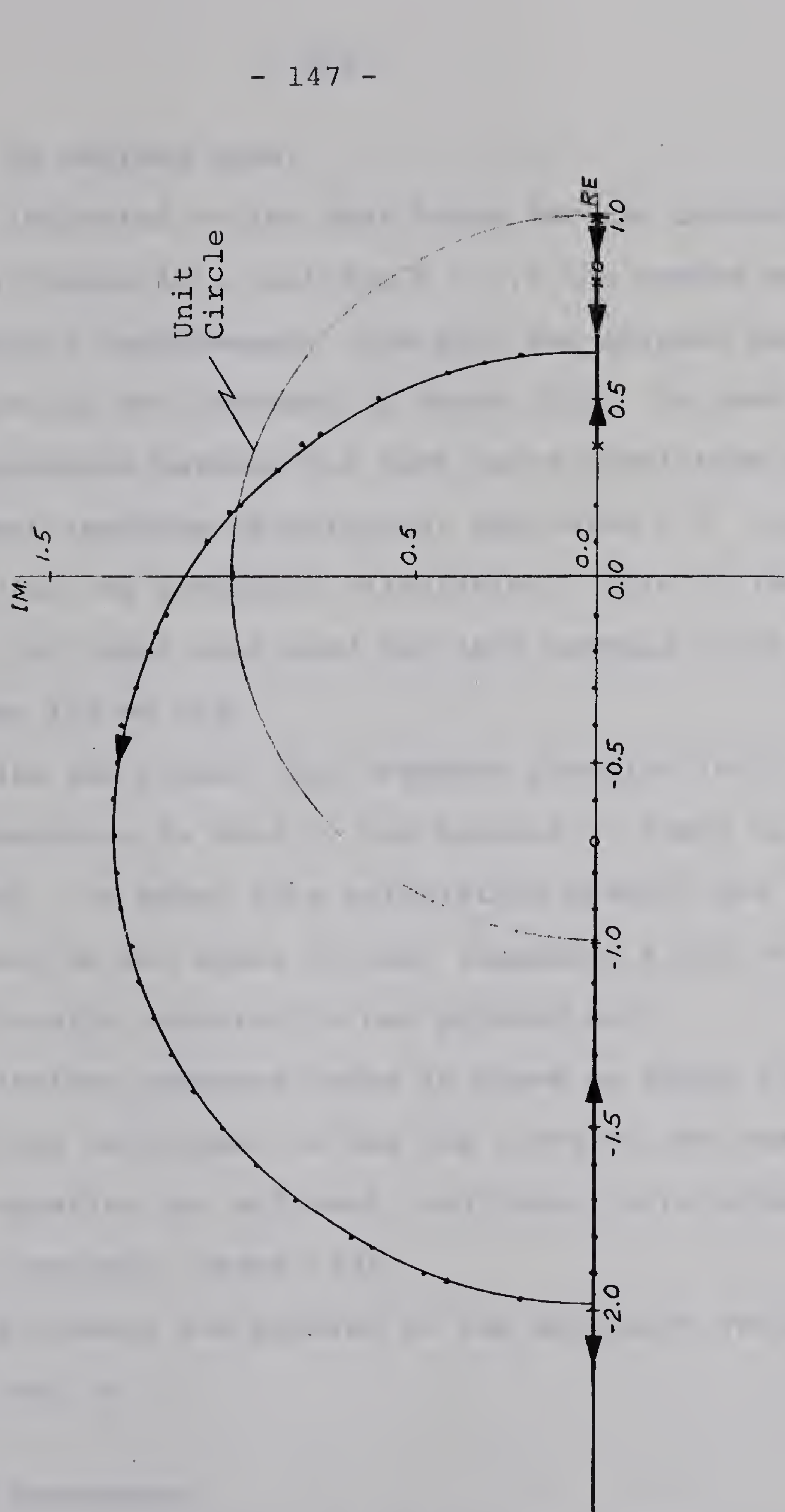
3. Let the controller zero be placed at Z = 0.834. The placing of this zero decides the integral time of the controller. As the zero is placed closer and closer to the pole, the less integral action there is in the system. This placing of the zero is done mainly by trial and error, no criteria has been followed.

4. Now

$$\frac{\theta_o(Z)}{I(Z)} = \frac{KD(Z)G(Z)}{1 + KD(Z)G(Z)}$$

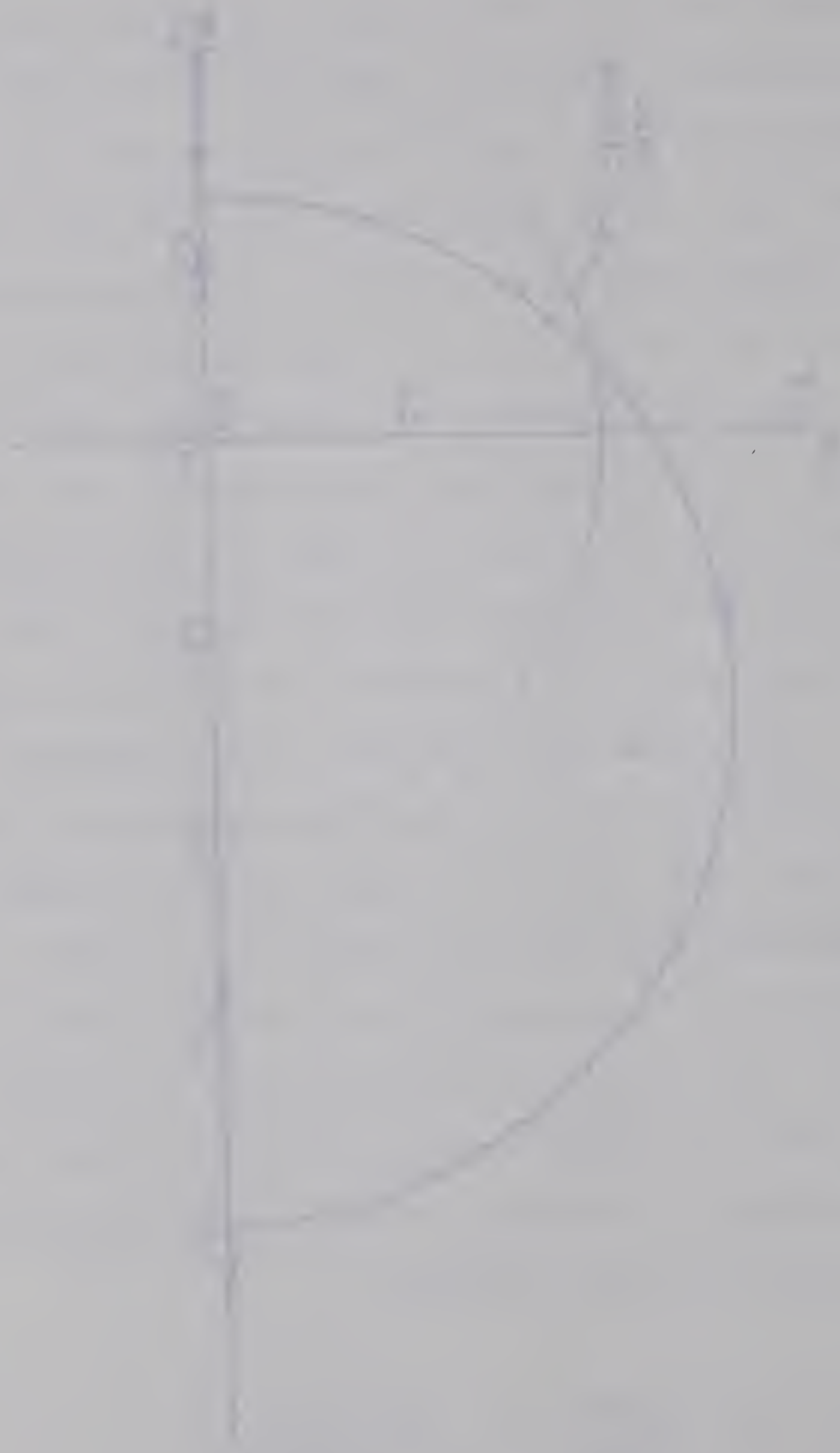
D(Z) has been designated as

$$D(Z) = \frac{Z - 0.834}{Z - 1.0}$$



Graph 12

Z-Plane Root Locus



K must be decided upon.

It was indicated by the Root Locus for the controlled system, (Graph 12), that for K = 1.0 the system would be slightly underdamped. The gain was printed out on paper but is not included in Graph (12). To test the correspondence between the root locus prediction and the transient response calculation, the value K = 1.0 was chosen for the transient calculation. This is the first trial, the input data used for this example is listed on pages 153 to 158.

5. Calculate the closed loop transfer function in Z.

   This operation is done by the machine if there is unity feedback. To enter this calculation branch, the flag LOOP must be set equal to one, (Appendix A.3.1, A.5.1). This transfer function is not printed out.

6. The transient response curve is shown as Graph (13).

   For a loop gain equal to one the roots of the characteristic equation are all real, and there is no overshoot in the response, Graph (13).

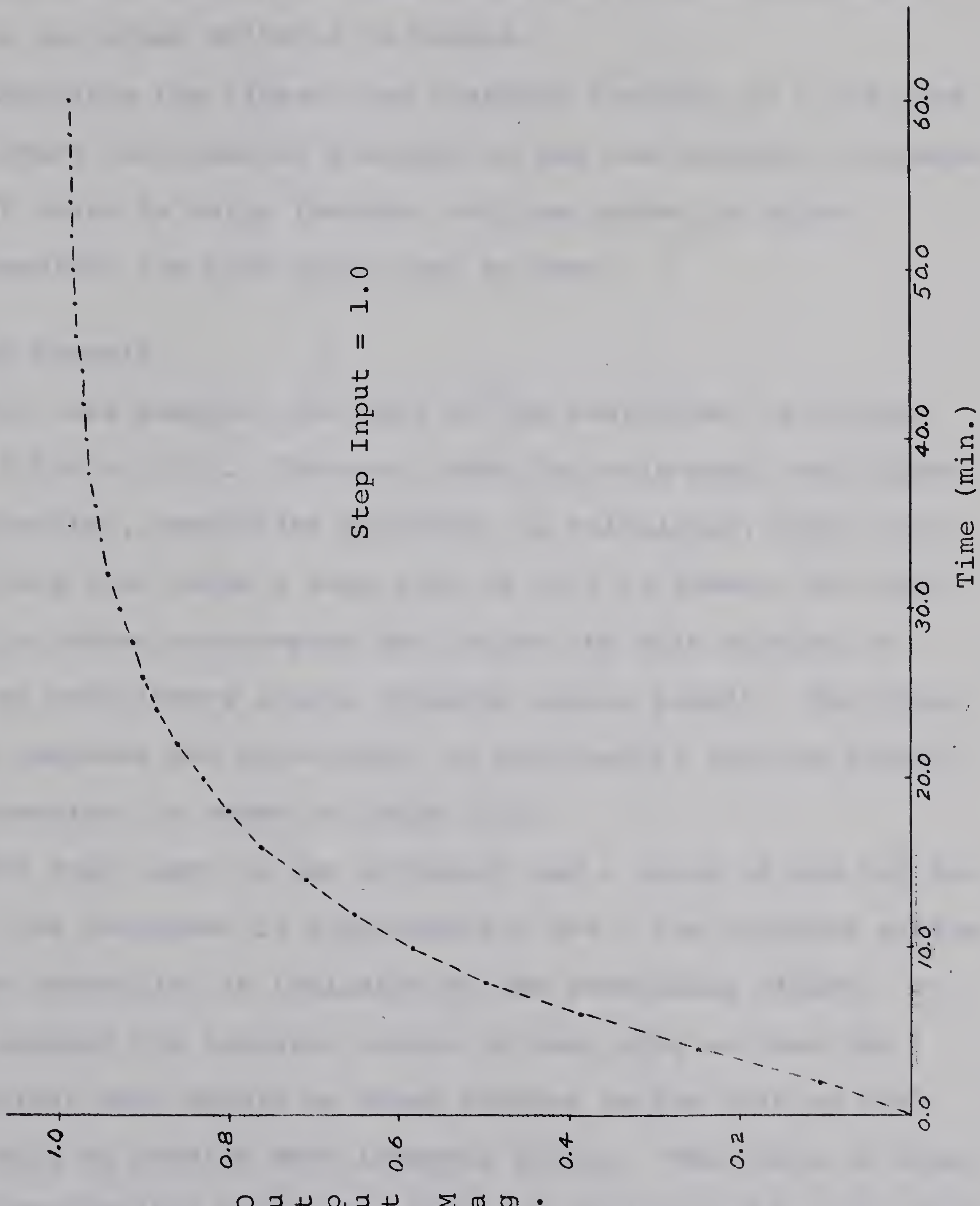
   The disturbance was applied to the set-point variable and was a step of 1.

Summary of Procedure:

1. Obtain the Z-transform of the process and hold, that is obtain $HG_p(Z)$ using the C.S.A. program.

Graph 13

Transient Response Curve



2. Obtain the Z-transform of the selected digital controller. This is formed by the user using the Z-plane root locus or any other criteria he wishes.
3. Calculate the closed loop transfer function in Z and then invert the transfer function to get the transient response. If there is unity feedback and the system is error-sampled, the LOOP option may be used.

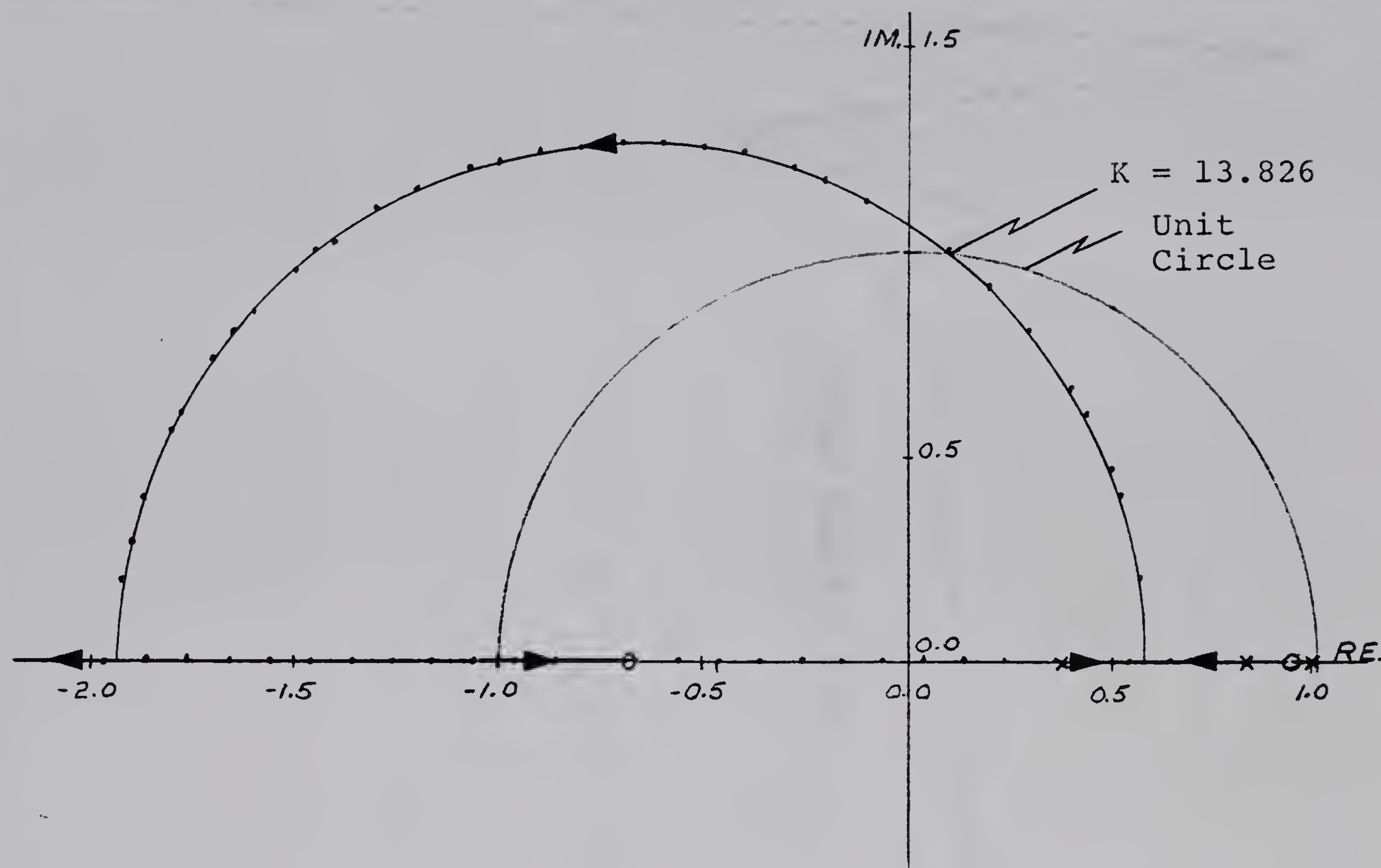
Second Example:

For this example, the zero of the controller is changed from 0.834 to 0.95. The root locus for this open loop transfer function, controller included, is calculated, Graph (14). From this root locus a loop gain of 12.0 is chosen. For this gain the roots are complex but inside the unit circle, so that an oscillatory stable response should result. The transient response was calculated, as previously, and the resulting transient is shown in Graph (15).

The step input to the set-point had a value of one and for this, the overshoot is approximately 30%. The integral action of the controller is indicated by the decreasing offset. In this example the integral action is very slow so that the controller zero should be moved further to the left on the real axis to provide more integral action. When this is done, the corresponding root locus should be calculated in case the gain of 12.0, which was used here, be too high for the modified system. The input data for this problem is listed on pages 159 to 161.

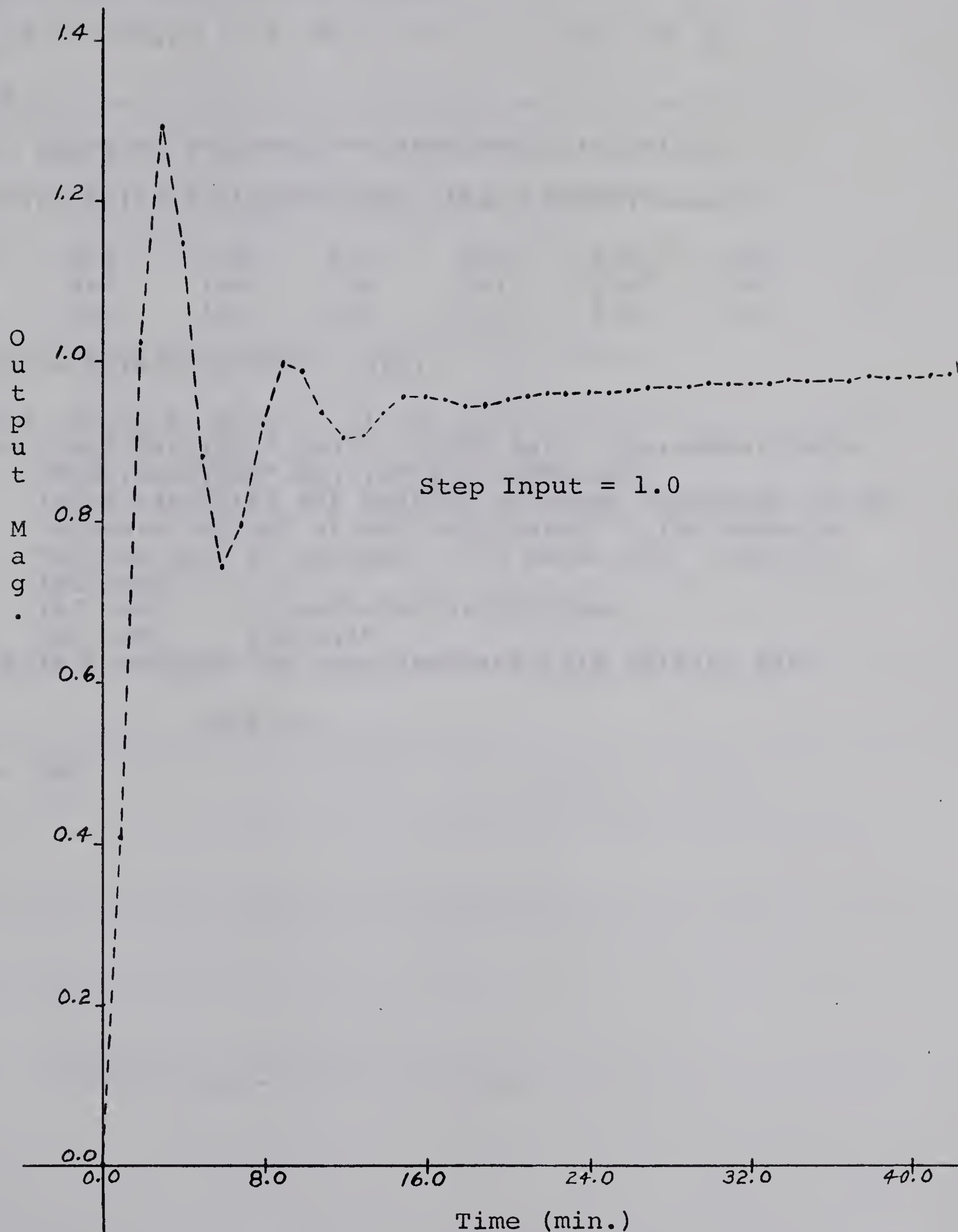
Graph 14

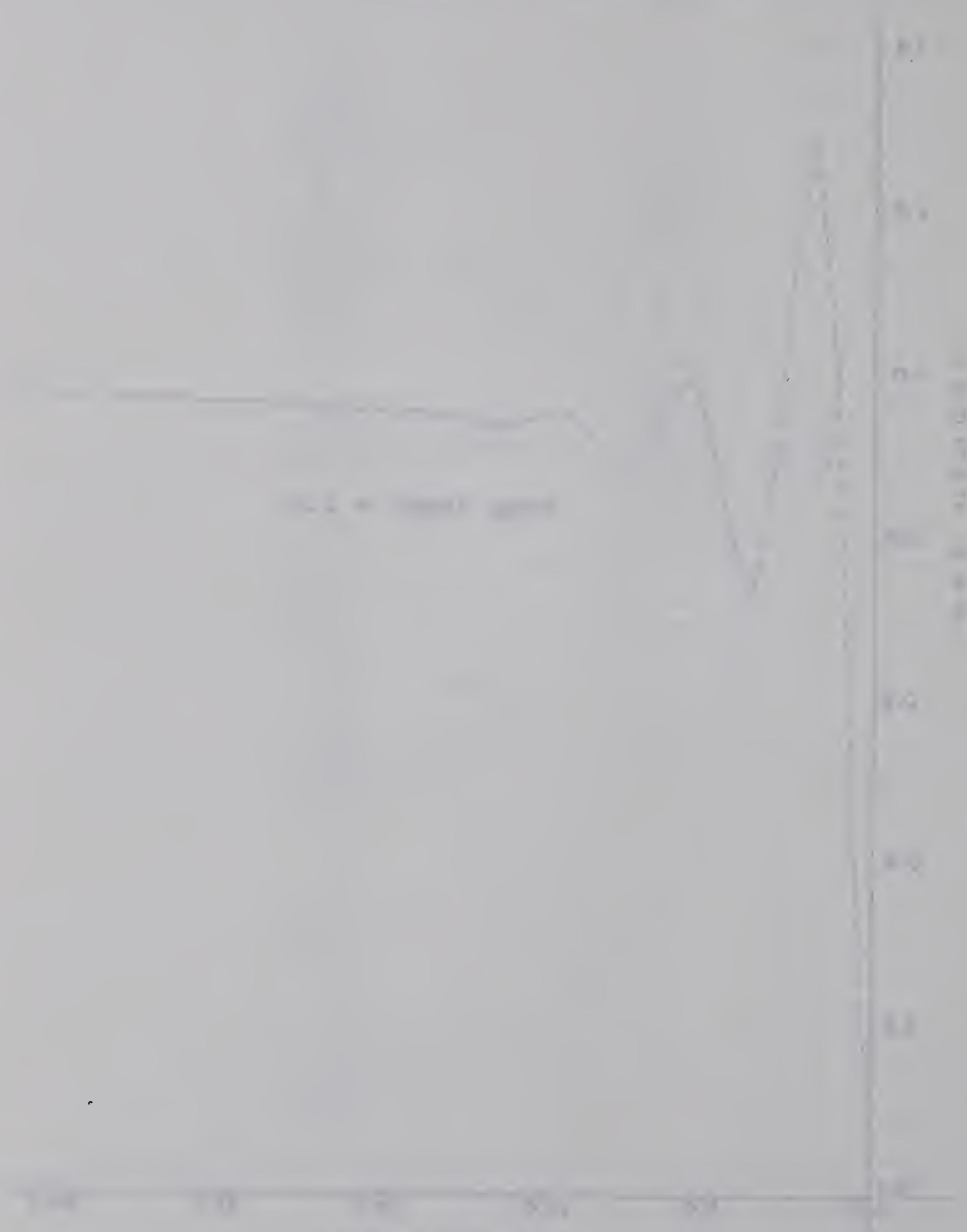
Z-Plane Root Locus



Graph 15

Transient Response Curve





INPUT DATA

CCNTRCL CARC CATA (FORMAT 24I3)

24 12 1 C-10 1 4 1 1 0 0 -2 0100 0 2

TITLE

RCOT LOCUS OF SYSTEM WITH SAMPLE-HOLD INSERTED.

A-VECTCR ENTRIES (COEFFICIENT DATA. FORMAT 9E8.5)

-C.1	0.2	1.0	0.0	2.0	4.C	1.0	
0.2	1.C	1.0	1.0	0.1	-1.0	1.0	0.2
1.0	1.C	1.0	0.0	1.0	C.C	1.0	

M-VECTCR ENTRIES (FORMAT 24I3)

2 1 1 0 2 C -1 4 1 1 1 -1

OTHER FLAGS AND INPUT DATA. THESE ARE, NYQ,OMEGA,DOMEG,
CMEGF,NBOC,NZRT,DT,T,FMT(1),STEP,LCOP.
THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
FCLLCWING FORMAT IF NCT APPLICABLE TC THE PROBLEM
NOTHING NEEC BE ENTERED. THE CARDS MUST STILL BE
INCLUDEC.

1ST CARC I5,3E10.5,2I5,2E10.5,A6

2NC CARC E1C.5,I5

FOR THIS PROBLEM THE SUPPLEMENTARY CATA ENTRIES ARE,

NZRT=1

DATA ENC

```
F1  NO. OF A-VECTOR TERMS                                      N = 24

F2  NO. OF M-VECTOR TERMS                                      N = 12

F3  NO. OF RUNS TO BE MADE.                                    N =  1

F4  ONE OF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                   N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE OF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS OF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER                   N =-10
       SYSTEM BUT NO Z-FORM    N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER OF S                                         N =  1

F7  HIGHEST POWER OF Z                                         N =  4

F8  NUMBER OF VALUES ASSIGNED TO T                             N =  1

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)             N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                           N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                            N =  0

F12  REPORT HEADING OPTION  (N=+,-2)                           N = -2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                                N =  0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS           N =100

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
       N=DEGREE OF RESULTING Z-FORM DENOMINATOR             N =  2
                +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                      N = -0

F19  REAL PART  N=-10K                                      N = -0


                      M-VECTOR DATA

                      NO FEEDBACK

FORWARD LOOP NO.  1

      NO. OF TERMS IN NUMERATOR N =  2
        DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
              C -1

      NO. OF TERMS IN DENOMINATOR  N =  4
        DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
              1  1  1 -1
```

INPUT DATA

```
CONTROL CARD DATA (FORMAT 24I3)
 24 12  1  C-10  1  4  1  1  0  0 -2  01C0  0  2

TITLE
 TRANSIENT RESPONSE.  DIGITAL CONTROLLER INCLUDED.

A-VECTOR ENTRIES (COEFFICIENT DATA.  FORMAT 9E8.5)

-C.1     C.2      1.0      0.0      2.0      4.C      1.0
0.2      1.C      1.0      1.0      0.1      -1.0     1.0      0.2
1.0      1.C      1.0      0.0      1.0      0.C      1.0

M-VECTOR ENTRIES (FORMAT 24I3)

  2  1  1  C  2  C -1  4  1  1  1 -1

OTHER FLAGS AND INPUT DATA.  THESE ARE,  NYQ,OMEGA,DOMEG,
      OMEGF,NBOD,NZRT,DT,T,FMT(1),STEP,LOOP.
      THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
      FOLLOWING FORMAT IF NOT APPLICABLE TO THE PROBLEM
      NOTHING NEED BE ENTERED.  THE CARDS MUST STILL BE
      INCLUDED.
      1ST CARD      I5,3E10.5,2I5,2E10.5,A6
      2ND CARD      E1C.5,I5
  FOR THIS PROBLEM THE SUPPLEMENTARY DATA ENTRIES ARE,
  NZRT=1,  DT=1.C,  T=60.0,  FMT(1)=MIN.  STEP=1.0,  LOOP=1

M1 VECTOR (FORMAT 10I3)

  1  1  1  1

A1 VECTOR (FORMAT 9E8.5)

-0.834  1.C     -1.0     1.0

          FOR INFORMATION ON LOOP = 1 DATA INPUT
            REFER TO SUBROUTINE AMALG, APPENDIX
                         A-
DATA  END
```

```
F1  NO. OF A-VECTOR TERMS                                    N = 24

F2  NO. OF M-VECTOR TERMS                                    N = 12

F3  NO. OF RUNS TO BE MADE.                                  N =  1

F4  ONE OF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                 N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE OF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS OF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER                 N =-10
       SYSTEM BUT NO Z-FORM   N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER OF S                                       N =  1

F7  HIGHEST POWER OF Z                                       N =  4

F8  NUMBER OF VALUES ASSIGNED TO T                           N =  1

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)           N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                         N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                          N =  0

F12  REPORT HEADING OPTION  (N=+,-2)                         N = -2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                              N =  0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS         N =100

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
       N=DEGREE OF RESULTING Z-FORM DENOMINATOR           N =  2
                 +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                    N = -0

F19  REAL PART  N=-10K                                    N = -0


                   M-VECTOR DATA


                   NO FEEDBACK

FORWARD LOOP NO.  1

    NO. OF TERMS IN NUMERATOR N =  2
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            0 -1

    NO. OF TERMS IN DENOMINATOR  N =  4
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  1  1 -1
```

INPUT DATA

```
CCNTRCL CARC CATA (FCRMAT 24I3)
 24 12  1  C-10  1  4  1  1  0  0 -2  01C0  0  2

TITLE
 CIGITAL CCNTRCLLER CESIGN

A-VECTCR ENTRIES (COEFFICIENT DATA.  FCRMAT 9E8.5)

-0.1     0.2      1.0      0.0      2.0      4.C      1.0
0.2      1.C      1.0      1.0      0.1      -1.0     1.0      0.2
1.C      1.C      1.0      0.0      1.0      C.C      1.0

M-VECTCR ENTRIES (FORMAT 24I3)

  2  1  1  C  2  C -1  4  1  1  1 -1

CTHER FLAGS ANC INPUT CATA.  THESE ARE,  NYQ,OMEGA,DOMEG,
      CMEGF,NBOC,NZRT,DT,T,FMT(1),STEP,LOOP.
      THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
      FCLLCWING FORMAT IF NCT APPLICABLE TC THE PROBLEM
      NOTHING NEEC BE ENTERED.  THE CARDS MUST STILL BE
      INCLUCEC.
      1ST CARC      I5,3E10.5,2I5,2E10.5,A6
      2NC CARD      E1C.5,I5
  FOR THIS PROBLEM THE SUPPLEMENTARY CATA ENTRIES ARE,
 NZRT=1, CT=1.0, T=60.0, FMT(1)=MIN., STEP=1.0, LOOP=1

M1 VECTCR (FORMAT 10I3)

  2  1  C  1  1

A1 VECTCR (FORMAT 9E8.5)

-C.95    1.C      12.0     -1.0     1.0

          FCR INFORMATION ON LOOP = 1 DATA INPUT
            REFER TO SUBROUTINE AMALG, APPENDIX
                         A-4
DATA  END
```

```
F1  NO. OF A-VECTOR TERMS                                   N = 24

F2  NO. OF M-VECTOR TERMS                                   N = 12

F3  NO. OF RUNS TO BE MADE.                                 N =  1

F4  ONE OF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE OF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS OF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER                N =-10
       SYSTEM BUT NO Z-FORM    N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER OF S                                      N =  1

F7  HIGHEST POWER OF Z                                      N =  4

F8  NUMBER OF VALUES ASSIGNED TO T                          N =  1

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)          N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                        N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                         N =  0

F12  REPORT HEADING OPTION  (N=+,-2)                        N = -2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                                    N =   0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS               N =100

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
        N=DEGREE OF RESULTING Z-FORM DENOMINATOR                N =   2
                    +N  RECT LOCUS POINTS

F17  B-MATRIX  YES N=1                                          N =  -0

F19  REAL PART  N=-10K                                          N =  -0


                    M-VECTOR DATA


                    NO FEEDBACK


FORWARD LOOP NO.  1

      NO. OF TERMS IN NUMERATOR N =   2
        DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
              0 -1

      NO. OF TERMS IN DENOMINATOR  N =   4
        DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
              1  1  1 -1
```

APPENDIX A.9.7

DIRECT REPLACEMENT OF A CONTINUOUS CONTROLLER BY A DIGITAL CONTROLLER

Purpose:

To illustrate the method outlined in Section (7.4.2) for the design of digital controllers.

Transfer Functions:

Continuous System with Controller

$$G_c(s)G_p(s) = \frac{s+0.1}{s(s+1.0)(s+0.2)}$$

Process Transfer Function

$$G_p(s) = \frac{0.1}{(s+1.0)(s+0.2)}$$

Process Transfer Function with Hold

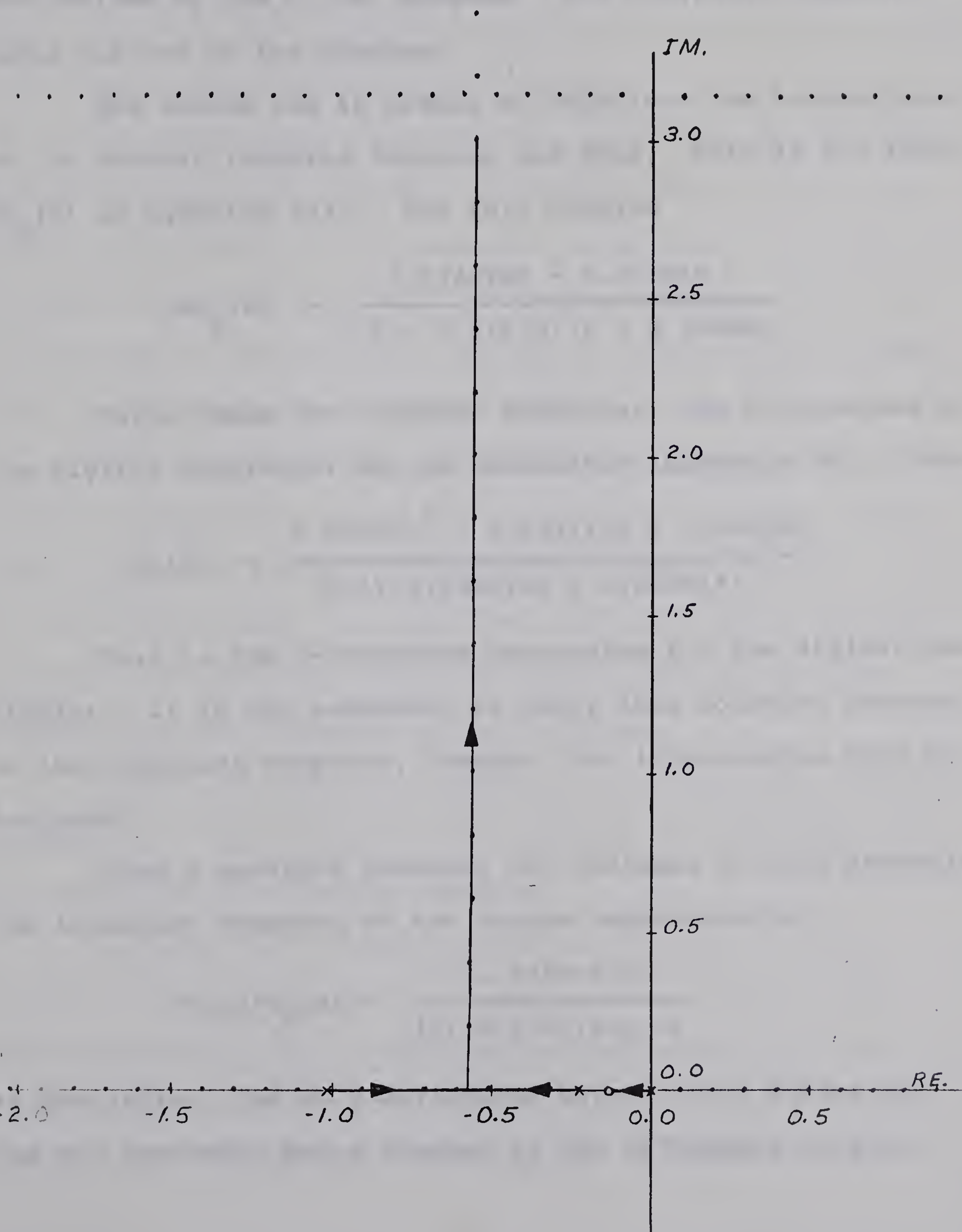
$$H_o(s)G_p(s) = \left(\frac{1 - e^{-sT}}{s}\right)\left(\frac{0.1}{(s+1.0)(s+0.2)}\right)$$

Outline:

The first step in the design is to derive the Z-transform of the continuous system with the continuous controller included. The root locus for this system (Graph 16), is also obtained so that a loop gain may be decided upon for the transient response calculation. The associated gains are not included on the graph. This Z-transform

Graph 16

s-Plane Root Locus for the Z-Transform of the Continuous System



$$Z\left[G_c(s)G_p(s)\right] = \frac{0.59555678Z^2 - 0.5391119Z - 0.000269284}{(Z - 0.81873076)(Z - 0.36787945)(Z-1)}$$

was derived by the C.S.A. program. The derivation constitutes one run of the program.

The second run is needed to calculate the Z-transform of the process transfer function and hold. This is the term $HG_p(Z)$ in equation (43). For this problem

$$HG_p(Z) = \frac{0.034278Z + 0.023014}{(Z - 0.81873)(Z - 0.36788)}$$

Using these two transfer functions, the Z-transform of the digital controller may be calculated (Equation 45). Thus,

$$D_c(Z) = \frac{0.59557Z^2 - 0.539112Z - 0.000269}{(Z-1)(0.034278Z + 0.023014)}$$

This is the Z-transform expression for the digital controller. It is not necessary to carry this solution through to the transient response, however, for illustration this is included.

From a previous problem, not included in this appendix, the transient response of the system represented as

$$G_c(s)G_p(s) = \frac{1.44(s+0.1)}{(s)(s+1.0)(s+0.2)}$$

is available. The only difference between this system and the one currently being treated is the difference in gain.

Changing the process transfer function numerator from 0.1 to 0.144 will effectively match the gains since a gain of 10 is already included in the digital controller transfer function. With the gain matching completed one more run is necessary to arrive at the transient response.

This particular value of gain was picked since a comparison was available, however, different values can be used without redesigning the controller. It is only necessary to multiply either the numerator of the controller or that of the process by the suitable constant to give the gain chosen from the Root Locus Plot, Graph (16).

The input data for each of the three runs is shown with the appropriate titles. The root locus for the process under continuous control (Graph 16) is shown. The root locus for the system under the equivalent digital control was calculated as a check for error but was the same as the continuous case and consequently is not shown.

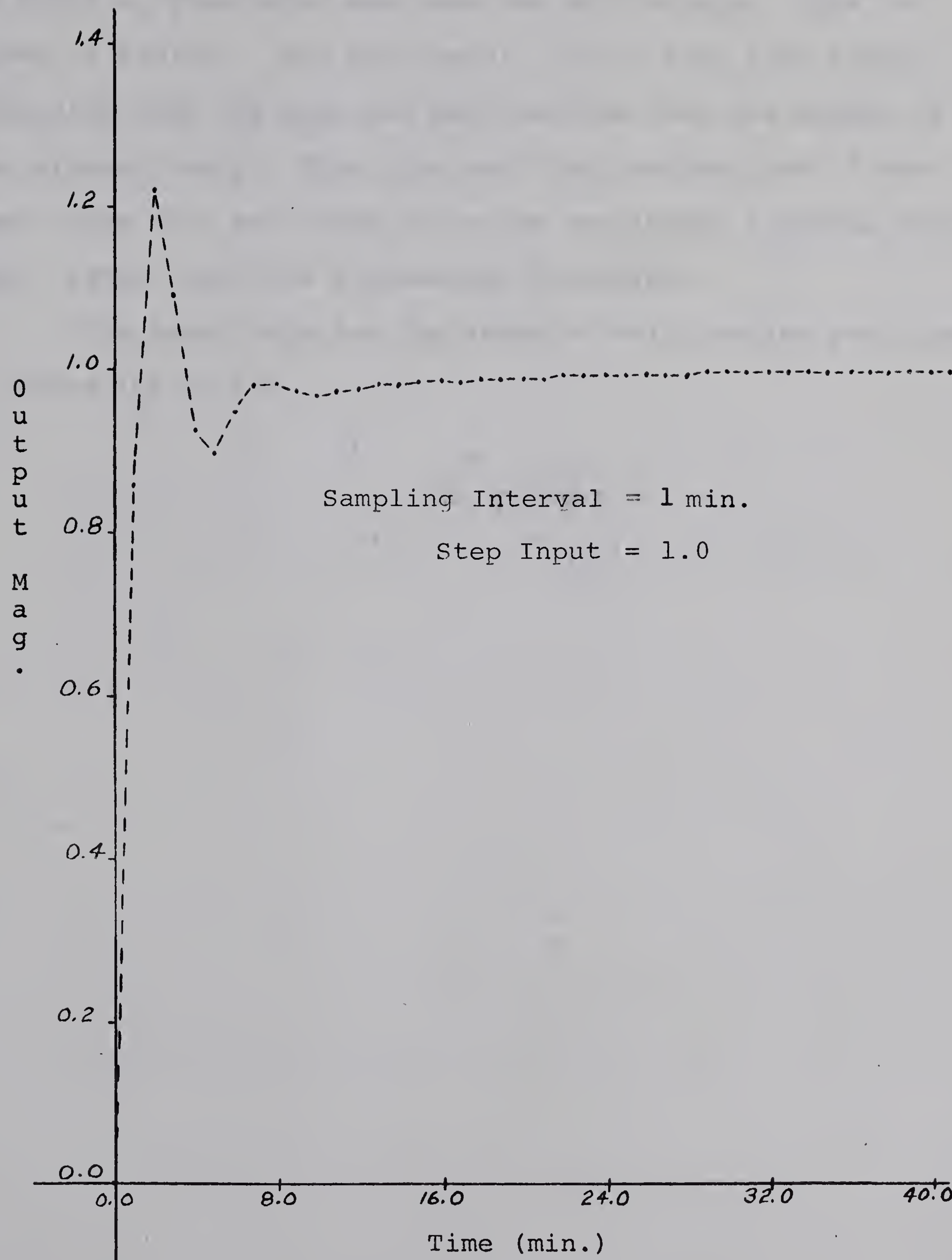
Graph (17) gives the results of the transient response calculation. This response was identical to that found for the continuously controlled system.

In Graph (16) a horizontal line is shown at Re(s) = 3.139. This line marks $w_s/2$ which is the sampling frequency divided by two. When mapping the s-plane into the Z-plane the mapping is initially carried out over a primary strip in the left half of the s-plane. The rest of the

Graph 17

Transient Response Continuous

Controller Replacement



s-plane is divided into an infinite number of periodic strips of width w_s which also map into the unit circle. This is shown in Kuo(12). The horizontal line in this plot simply indicates that the scan has been carried over the border of the primary strip. This line would not be obtained if the root locus in s was found using the continuous transfer function, rather than the Z-transform equivalent.

The input data for the steps in this problem are listed on pages 168 to 173.

INPUT DATA

CONTROL CARD DATA (FORMAT 24I3)

```
 24 12  1  C-10  1  3  1  0  0  0 -2  0 50  0  3
```

TITLE

Z-TRANSFORM OF CONTINUOUS SYSTEM

A-VECTOR ENTRIES (COEFFICIENT DATA. FORMAT 9E8.5)

```
-C.1    0.2     1.0     0.0     4.0     4.C     1.0
0.2     1.C     1.0     1.0     -1.0    1.C     5.0     1.0
0.72    C.2     1.0     1.0     1.0     0.C     1.0
```

M-VECTOR ENTRIES (FORMAT 24I3)

```
  3  1  1 -1  0  2  1  0  3  1  1  1
```

OTHER FLAGS AND INPUT DATA. THESE ARE, NYQ,OMEGA,DOMEG, OMEGF,NBOD,NZRT,DT,T,FMT(1),STEP,LOOP. THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE FOLLOWING FORMAT IF NOT APPLICABLE TO THE PROBLEM NOTHING NEED BE ENTERED. THE CARDS MUST STILL BE INCLUDED.

```
1ST CARD      I5,3E10.5,2I5,2E10.5,A6
2ND CARD      E10.5,I5
```

FOR THIS PROBLEM THE SUPPLEMENTARY DATA ENTRIES ARE,

NIL

DATA END

```
F1  NO. CF A-VECTCR TERMS                                    N = 24

F2  NO. CF M-VECTOR TERMS                                    N = 12

F3  NO. CF RUNS TO BE MADE.                                  N =  1

F4  ONE CF FCUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TC BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                 N =  0
       LCOP WITH UNITY FEEDBACK.  N=0
      -CTHERWISE  N=N

F5  ONE CF THREE OPTIONS SPECIFIED.
      -FCR RCOT LOCUS CF CONTINUOUS SYSTEM OR
       FCR A SYSTEM IN Z-FORM.  N=0
      -FCR RCOT LOCI PCINTS OF A ONE-SAMPLER                 N =-10
       SYSTEM BUT NO Z-FORM   N=1
      -FCR Z-TRANSFORM COMPUTED OR ROOT LOCI
       PCINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST PCWER OF S                                       N =  1

F7  HIGHEST POWER OF Z                                       N =  4

F8  NUMBER OF VALUES ASSIGNED TO T                           N =  1

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)           N =  1

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                         N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                          N =  0

F12  REPORT HEADING OPTION  (N=+,-2)                         N = -2
      UNUSUAL Z-FORM OPTICN  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                                   N =  0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS              N = 50

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
       N=DEGREE OF RESULTING Z-FORM DENOMINATOR                N =  3
                +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                         N = -0

F19  REAL PART  N=-10K                                         N = -0


                        M-VECTOR DATA


                        NO FEEDBACK


FORWARD LOOP NO.  1

    NO. OF TERMS IN NUMERATOR N =  2
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  0

    NO. OF TERMS IN DENOMINATOR  N =  3
      DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  1  1
```

INPUT DATA

```
CONTROL CARD DATA (FORMAT 24I3)

 24 12  1  0-10  1  4  1  1  0  0 -2  0100  0  2

TITLE

  ANALOGUE TO DIGITAL CONTROLLER

A-VECTOR ENTRIES (COEFFICIENT DATA.  FORMAT 9E8.5)

 -0.1    0.2     1.0     0.0     2.0     4.0     1.00
0.2     1.0     1.0     1.0     0.144   -1.0    1.0     0.2
1.0     1.0     1.0     0.0     1.0     0.0     1.0

M-VECTOR ENTRIES (FORMAT 24I3)

  2  1  1  0  2  0 -1  4  1  1  1 -1

OTHER FLAGS AND INPUT DATA.  THESE ARE,  NYQ,OMEGA,DOMEG,
      OMEGF,NBOD,NZRT,DT,T,FMT(1),STEP,LOOP.
      THESE VARIABLES ARE ENTERED AS ABOVE ACCORDING TO THE
      FOLLOWING FORMAT IF NOT APPLICABLE TO THE PROBLEM
      NOTHING NEED BE ENTERED.  THE CARDS MUST STILL BE
      INCLUDED.
      1ST CARD     I5,3E10.5,2I5,2E10.5,A6
      2ND CARD     E10.5,I5
  FOR THIS PROBLEM THE SUPPLEMENTARY DATA ENTRIES ARE,
   DT=1.0 T=60.0, FMT(1)=MIN., STEP=1.0, LOOP=1

M1 VECTOR (FORMAT 10I3)

  1  2  2  1  1

A1 VECTOR (FORMAT 9E8.5)
-.000269-.539112.59557  .023014 .034278 -1.0    1.0

          FOR INFORMATION ON LOOP = 1 DATA INPUT
            REFER TO SUBROUTINE AMALG, APPENDIX
                         A-4

DATA  END
```

```
F1  NO. OF A-VECTOR TERMS                                        N = 24

F2  NO. OF M-VECTOR TERMS                                        N = 12

F3  NO. OF RUNS TO BE MADE.                                      N =  1

F4  ONE OF FOUR OPTIONS SPECIFIED
      -IF Z-TRANSFORM TO BE COMPUTED  N=0
      -TWO SAMPLER SYSTEM. N=N
      -ALL TRANSFER FUNCTIONS IN ONE FORWARD                     N =  0
       LOOP WITH UNITY FEEDBACK.  N=0
      -OTHERWISE  N=N

F5  ONE OF THREE OPTIONS SPECIFIED.
      -FOR ROOT LOCUS OF CONTINUOUS SYSTEM OR
       FOR A SYSTEM IN Z-FORM.  N=0
      -FOR ROOT LOCI POINTS OF A ONE-SAMPLER                     N =-10
       SYSTEM BUT NO Z-FORM   N=1
      -FOR Z-TRANSFORM COMPUTED OR ROOT LOCI
       POINTS FOR TWO-SAMPLER SYSTEM N=-(10+...)

F6  HIGHEST POWER OF S                                           N =  1

F7  HIGHEST POWER OF Z                                           N =  3

F8  NUMBER OF VALUES ASSIGNED TO T                               N =  1

F9  ROOT LOCI. -,+,OR BOTH FEEDBACK OPTION (1,3,0)               N =  0

F10 SCAN CONTROL  (N=0,1,-1) V+H,H,V                             N =  0

F11 MODIFIED Z-FORM OPTION  (N=0,1)                              N =  0

F12  REPORT HEADING OPTION  (N=+,-2)                             N = -2
      UNUSUAL Z-FORM OPTION  (N=1,2)
```

```
F13  LOCI OPTION,USUALLY N=0                                  N =   0

F14  TERMS IN SERIES FOR G*(S),N=0 GIVES 19 TERMS             N =100

F16  IF Z-TRANSFORM TO BE COMPUTED  N.NE.0
      N=DEGREE OF RESULTING Z-FORM DENOMINATOR                N =   2
                  +N  ROOT LOCUS POINTS

F17  B-MATRIX  YES N=1                                        N =  -0

F19  REAL PART  N=-10K                                        N =  -0


                        M-VECTOR DATA


                        NO FEEDBACK


FORWARD LOOP NO.  1

     NO. OF TERMS IN NUMERATOR N =  2
       DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            0 -1

     NO. OF TERMS IN DENOMINATOR  N =  4
       DEGREES OF THESE TERMS   -VE INDICATES Z-FORM
            1  1  1 -1
```